CHAPTER I

ORTHOGONAL (RECTANGULAR) PROJECTING OF GEOMETRICAL OBJECTS

In engineering practice the projection method is used to construct projections of objects on the plane. Drawings obtained are called projection drawings. For making drawings of technical forms the orthogonal projection is used. It means that a projecting straight line passes through each point of geometrical object, perpendicular to the projection plane. The point of its intersection with this plane is considered to be the orthogonal projection of the point (Fig. 1)

§ 1. Invariant Properties of Orthogonal Projecting

Properties of geometrical objects, which remain unchanged in projecting, are called independent or invariant with respect to the projection way chosen.

1. The projection of the point is a point (Fig. 1)

$$M \to M_1 .$$

2. The projection of the straight line is a straight line (Fig. 1)

$$l \to l_1 .$$

If a straight line is perpendicular to the projection of the plane, then its projection becomes the point (Fig. 2)

$$l(MN) \perp \Pi_1 \Rightarrow l(MN) \to l_1 = M_1 = N_1 .$$

3. If the point belongs to the line, then the projection of the point belongs to the projection of the line (Fig. 1)

$$C \in l \Rightarrow C_1 \in l_1 .$$

C o n s e q u e n c e from 2 and 3. In order to construct the projection of the straight line it is necessary and enough to construct projections of two points belonging to it (Fig. 1)

$$l(A \in l \wedge B \in l) \Rightarrow l_1(A_1 \in l_1 \wedge B_1 \in l_1) .$$

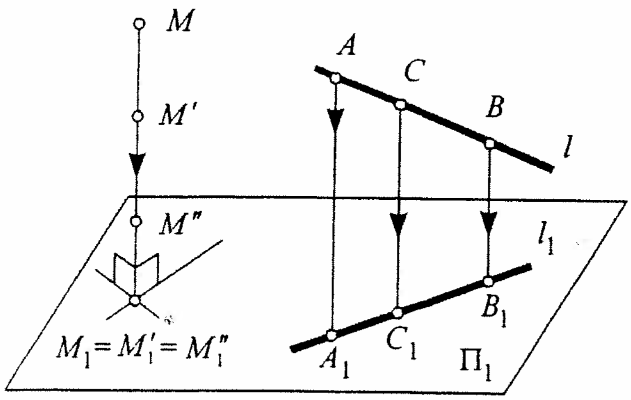


Fig. 1

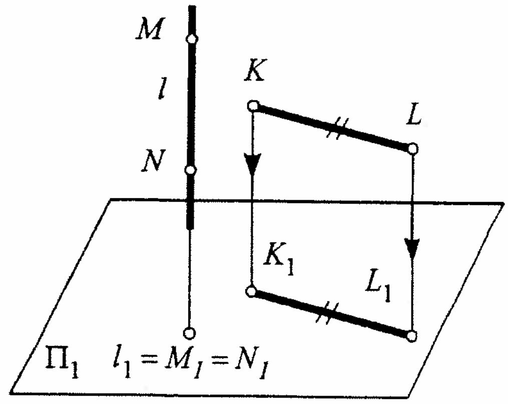


Fig. 2

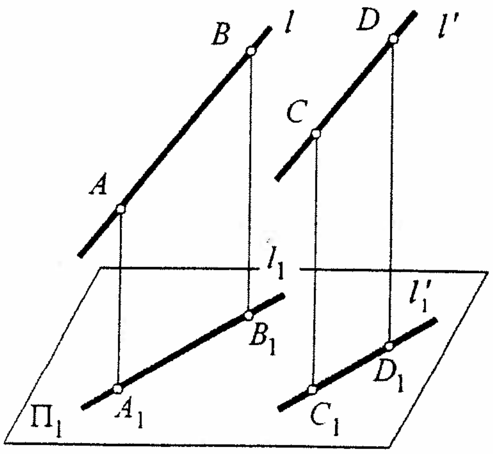


Fig. 3

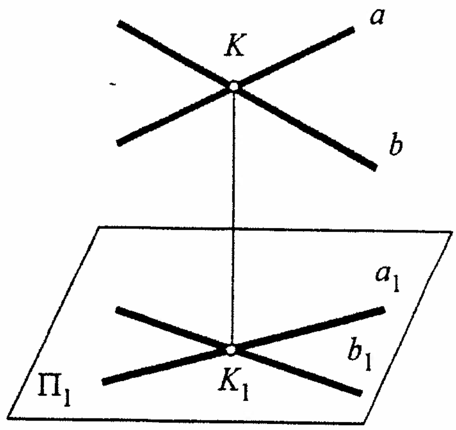


Fig. 4

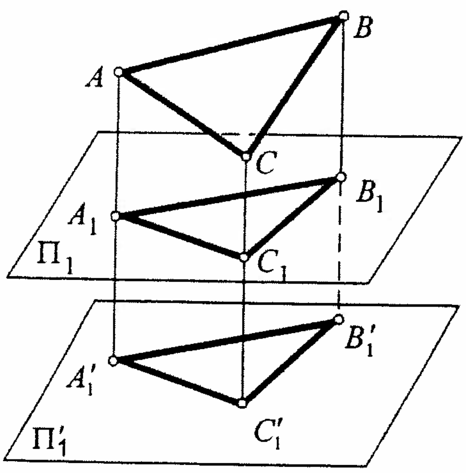


Fig. 5

4. Projections of parallel lines are parallel (Fig. 3)

$$l \parallel l' \Rightarrow l_1 \parallel l_1' \ .$$

5. If a plane geometrical object in parallel to the plane of projections, then the projection of this object on the plane of projections is congruent to the object itself:

$$\Phi \parallel \Pi_1 \Rightarrow \Phi_1 \cong \Phi \ .$$

For instance, if the line-segment [KL] is parallel to the plane of projections, its projection on the given plane of projections will be congruent to the line-segment itself (Fig. 2)

$$[KL] \parallel \Pi_1 \Rightarrow [K_1L_1] \cong [KL] \ .$$

6. The point of intersection lines is projected into the point of intersection of their projections (fig. 4)

$$(K = a \cap b) \rightarrow (K_1 = a_1 \cap b_1) \ .$$

7. The projection of a geometrical object is not changed with parallel movement of projection planes (Fig. 5).

The above method of projecting onto one plane of projections enables to solve the p r i m a l p r o b l e m: having an object to construct its projection. This problem can always be defined. In fact, each point of an object has only one projection since the projecting line intersects the plane of projection in one point (Fig. 2).

In practice it is necessary to be skilled not only to construct drawings but also to read them, i.e. to explain an object by means of the drawing. Imaginary representation of an object shape through its projection is a r e v e r s e p r o b l e m which can't always be defined: if one projection of the point is given (Fig. 2), it doesn't determine the locus of the point in space as it is in fact the projection of a number of points belonging to the projecting line. Therefore a single projection drawing of an object is unreversible.To get a reversible drawing an object is projected onto two, three and more planes relative to the complexity of an object shape.

§ 2. Complicated Drawings of Geometrical Objects

According to the loci theory a geometric object is determined as any locus of points. The simplest geometrical objects are a point, a line and a plane.

2.1. Point. Projection of the Point on Two and Three Planes of Projections

The system of three mutually perpendicular planes is adopted (Fig. 6):

Π_1 — the horizontal plane of the projection;

Π_2 — the frontal plane of the projection (located in front of the viewer);

Π_3 — the profile plane of the projection.

Lines of their intersection x_{12}, y_{13}, z_{23} are axes of projections.

Let's construct orthographic projections of the space point A on the plane Π_1, Π_2 and Π_3. The distance $|AA_1|$ from the point A to the plane Π_1 is called the *height* of the point A, while the distance $|AA_2|$ to the plane Π_2 is the *depth* of the point A, the distance $|AA_3|$ is the *latitude* of the point A.

If the projection planes Π_1, Π_2 and Π_3 are assumed to be coordinate planes of the Descartes system of coordinates, then the distances between the point and the projection plane are represented in a certain scale as coordinates of the point A: x, y, z. According to the point's coordinates the projection can always be constructed and according to the given projections its coordinate can be determined (see Fig. 6 and Fig.7). It is obvious that any two projections of the point A determine its locus in space (Fig. 6). In most cases to define the shape and the size of an object it is required to construct its projections on three planes. Let's consider the construction of a three-view drawing.

A space model of projection planes is inconvenient for practical usage as on the planes Π_1 and Π_3 the form and the sizes of geometrical projections are distorted. Therefore, a plane model is used in practice. In order to transfer from the space model to the plane one, the planes Π_1 and Π_3 should be brought into alignment with the fixed plane Π_2 by rotating around the axes x_{12} and z_{23}. Construction of the plane model for the point A is shown in Fig. 6 and Fig. 7.

The combination of two and more interconnected orthogonal projections of a geometric figure located on one plane is called a complicated drawing, i.e. a complex consisting of several projections. Conditions of communication between point projections on the complicated drawing are the following:

1) horizontal and frontal projections of the point belong to one vertical communication line;

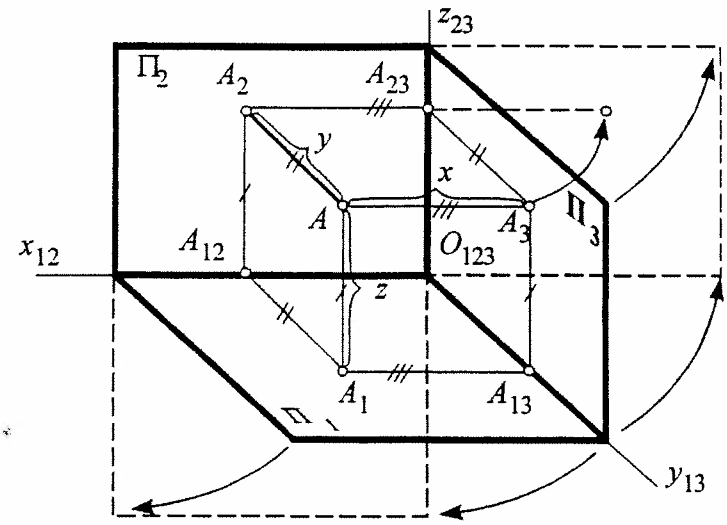


Fig. 6

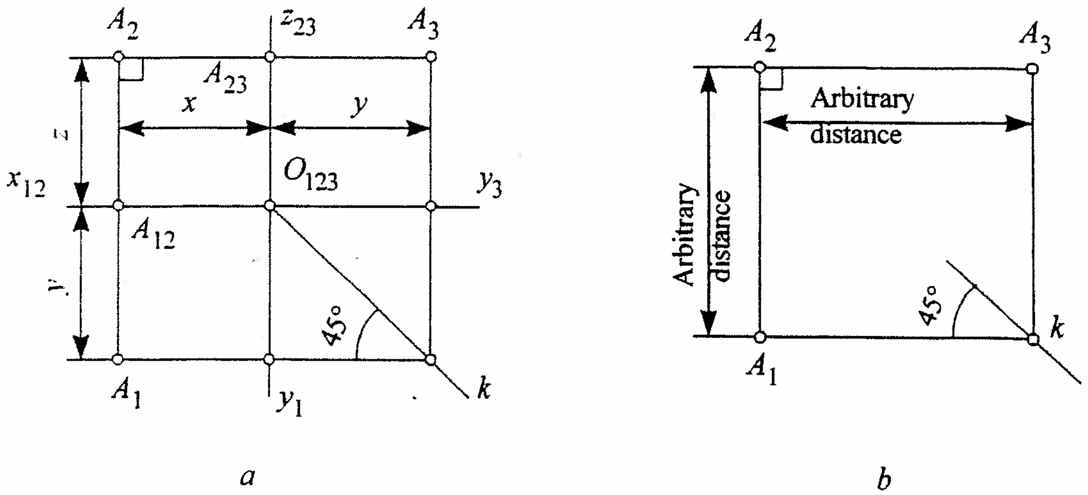


Fig. 7

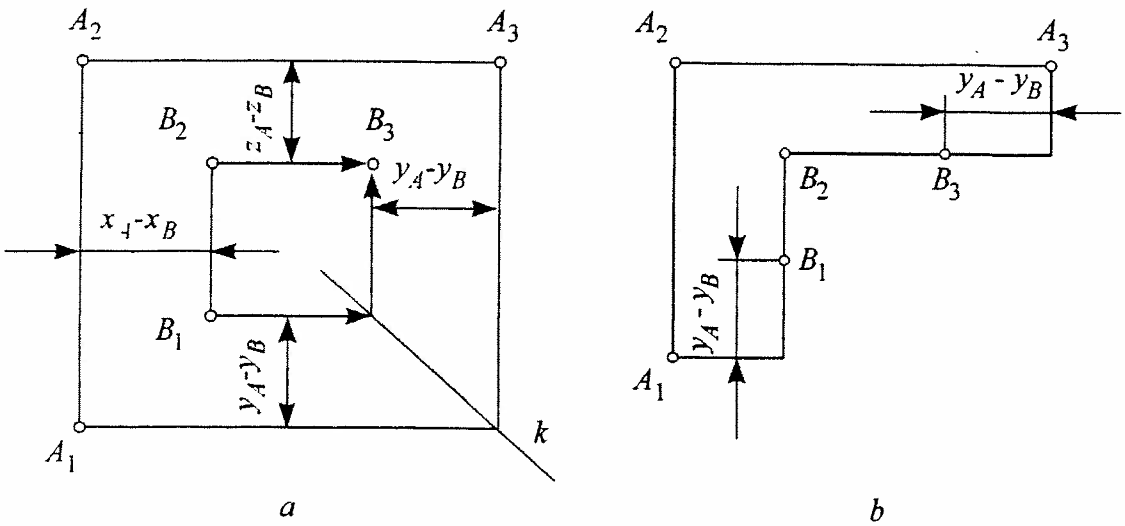


Fig. 8

2) frontal and profile projections of the point belong to one horizontal communication line;

3) horizontal and profile projections of the point belong to a broken communication line, the vertex of which belongs to the *constant line k* of the drawing named *a refraction line* (the line k is the bisector of the right angle formed by the broken communication line).

In engineering practice the non-axis method of making drawings is commonly used. In this case projection planes are not fixed in space, the axis of projections is not defined and marked on the drawing (Fig. 7, *b*). This method is based on the property 7 of the orthographic projecting (a geometrical projection of an object is not changed with the parallel movement of the projection plane). Conditions of connections between point projections remain on the поп-axis complicated drawing.

P r o b l e m. Given: the system of interconnected points A (A_1, A_2) and B (B_1, B_2) (Fig. 8). Required: to construct projections A_3 and B_3 of the given points.

Assuming the point A to be the base one, the profile projection A_3 is taken on the horizontal communication line at random. Construct the refraction line k, then the profile projection B_3 on account of conditions of communication between point projections on the complicated drawing (Fig. 8, *a*).

In a non-axis way of projecting point coordinates x, y, z become undefined. On the drawing (Fig. 8, *a*) differences between point coordinates A and B are given, which are not dependent on the position of projection planes and can be used for the construction of projections.

In solving this problem the difference of coordinates $|y_A - y_B|$ can be used to construct the profile projection of the point B (Fig. 8, *b*). The profile projection of the base point A is prescribed arbitrarily on the horizontal communication line. Draw then the horizontal connection line from the point B_2 and lay off on it leftward from the point A_3 the difference $|y_A - y_B|$, measured on the horizontal projection plane .

In practice in constructing the third projection of an object, axes, symmetry planes and other base planes of an object are assumed to be the basis of the distance reading (Fig. 9).

2.2. Lines

A line is denoted as a trace of a point constantly moving in space, as a surface boundary and as a result of an intersection of two surfaces. Lines enable to denote the functional dependence between parameters of any object or process

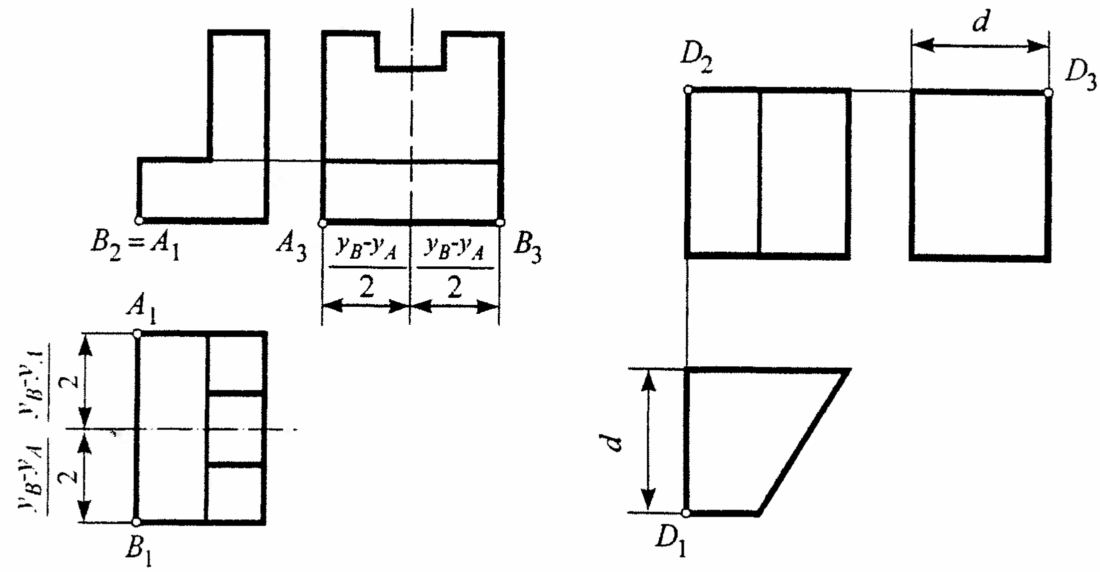


Fig. 9

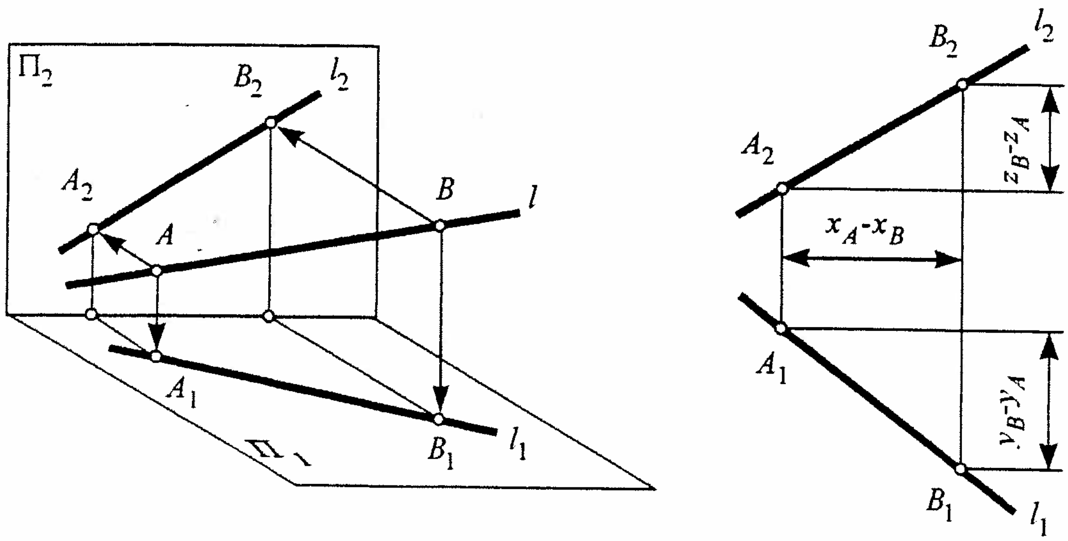


Fig. 10

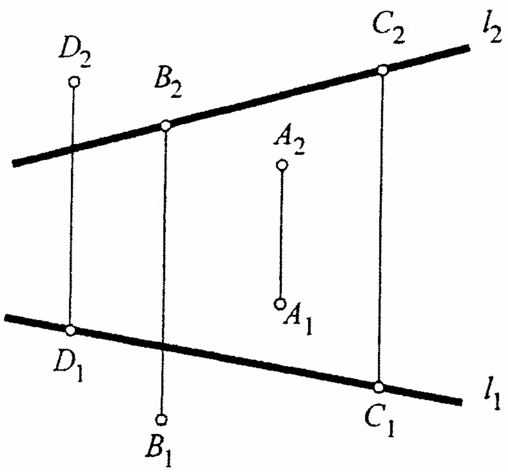


Fig. 11

in a graphic form and to construct the projection of an object on the drawing. Depending on the shape all lines can be classified into straight, broken and curves.

2.2.1. A Straight Line. Belonging of a Point to the Line. Lines of Particular Position

A projection of a straight line is generally a straight line, which can be drawn if projections of its two points are constructed.

The line of a general position is a straight line neither parallel nor perpendicular to any plane of projections. Two line projections of a general position determine its position in space, since each straight line point has two projections (Fig. 10).

The point can be located either on the line or outside. If the point belongs to the line, then the point projections belong to corresponding line projections. In Fig. 11 the point C belongs to the line $C \in l \Rightarrow C_1 \in l_1 \wedge C_2 \in l_2$, whereas points A, B and D do not belong to the line l: the point D is located above the line and the point B — in front of the line.

Lines of Particular Position.

1. ***Level lines***. The line parallel to one of the planes of projections is called the level line.

The horizontal $h \parallel \Pi_1$, $z_A - z_B = 0$ (Fig. 12, *a*); $h \parallel \Pi_1 \Rightarrow [A_1B_1] = |AB|$.

The angles α and β of the horizontal inclined to the planes Π_2 and Π_3 are projected on Π_1 in a true value.

The frontal $f \parallel \Pi_2$, $y_A - y_B = 0$ (Fig.12, *b*); $f \parallel \Pi_2 \Rightarrow [A_2B_2] = |AB|$.

The angles α and γ f the frontal inclined to the planes Π_1 and Π_3 are projected on Π_2 in a true value.

The profile line $p \parallel \Pi_3$, $x_A - x_B = 0$ (Fig. 12, *c*); $p \parallel \Pi_3 \Rightarrow [A_3B_3] = |AB|$.

The slopes α and β of the profile line to the planes Π_1 and Π_2 are projected on Π_3 in a true value.

2. ***Projecting lines***. A line perpendicular to one of the planes of projections is called a projecting line.

The horizontal-projecting line $g \perp \Pi_1$ (Fig. 13, *a*). The horizontal projection of this line degenerates into a point, the frontal and the profile projections are parallel to vertical communication lines.

The frontal projecting line $i \perp \Pi_2$ (Fig. 13, *b*). The frontal projection of this line degenerates into a point and horizontal and profile projections are parallel to vertical and horizontal communication lines, respectively.

The profile projecting line $q \perp \Pi_3$ (Fig. 13, *c*). The profile projection of this line degenerates into a point and frontal and horizontal projections are parallel to horizontal communication lines.

Points belonging to one projecting line are called competing points relative to the projection plane to which the line is perpendicular. For instance, points A and B are competitive relative to Π_1, points C and D are relative to Π_2, points M and N are relative to Π_3 (Fig. 13) and they are called the horizontal competing, the frontal competing and the profile competing points, respectively.

2.2.2. Curves

Curves are divided into two categories:

1) p l a n e curves, all points of which belong to one plane;

2) s p a c e curves, all points of which do not belong to one plane.

Curve projections in a general case are also curves. The construction of curve projections is in fact the construction of projections of some of its points.

The curves of the second order — ellipses, circles, parabolas and hyperbolas — can be constructed by intersecting the cone with the plane (Fig. 51). They are therefore called conic sections.

A circle is a plane curve of the second order, its orthographic projection can be represented as a circle, a line or an ellipse (Fig. 14). In Fig. 14, *b* the frontal projection of the circle, i.e. the ellipse is defined by the minor axis of the ellipse $A_2B_2 = d\cos\beta$ and by the major axis of the ellipse $C_2D_2 = d$.

Among regular space curves the most commonly used in practice is the cylindrical helical line formed as a result of a uniform helical motion of the point, its rotation around the axis and a translational motion parallel to this axis.

The value p of the point movement along the axis and corresponding to one revolution around the axis is called a step of the helical line. If the rotation axis i is perpendicular to one of the projection planes, the helical line is projected onto this plane as a circle, whereas in the plane parallel to the rotation axis it is projected as a sinusoid (Fig. 15).

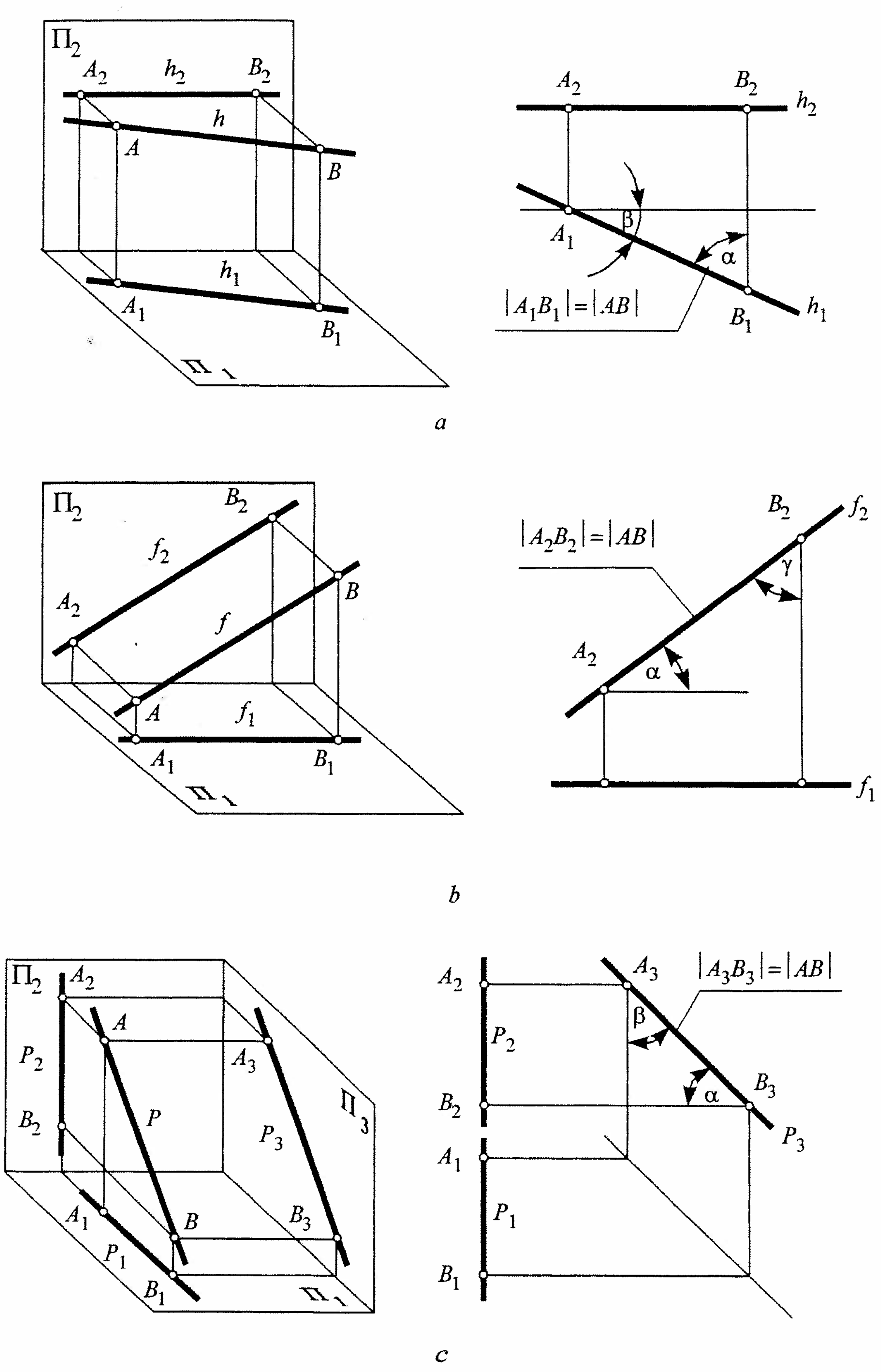
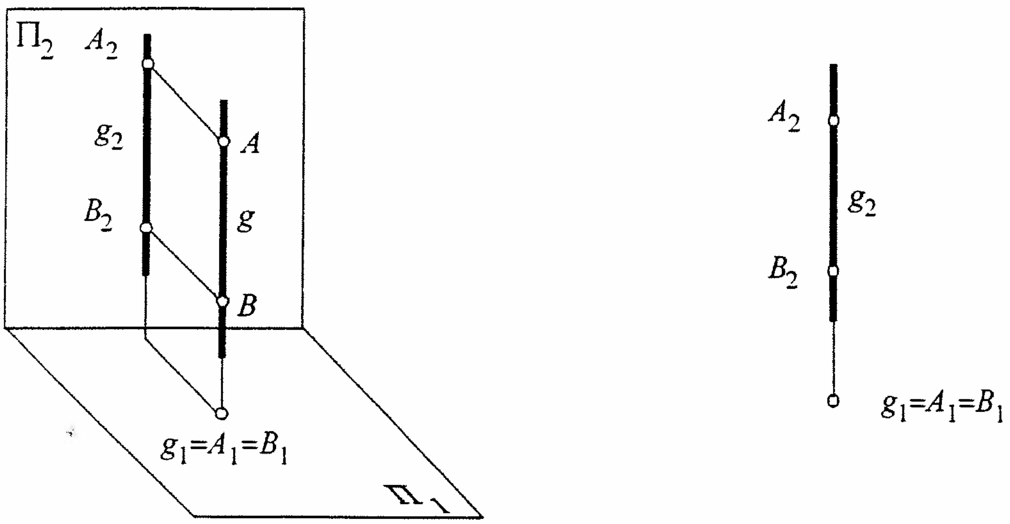
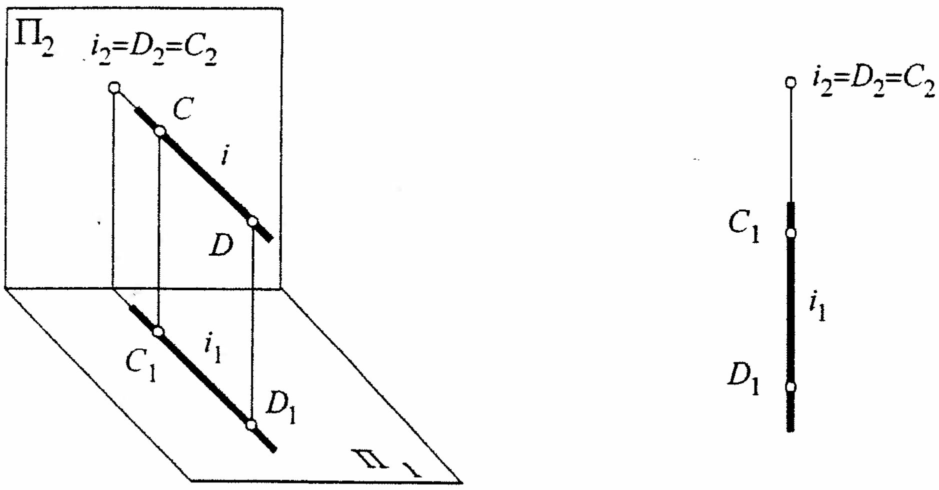

Π2
A2
h2
B2
h
A
B
h1
A1
B1
Π1
A2
B2
h2
β
A1
α
|A1B1|=|AB|
B1
h1
a
Π2
B2
f2
B
A2
f
A
f1
A1
B1
Π1
B2
f2
|A2B2|=|AB|
γ
A2
α
f1
b
Π2
A2
A
P2
A3
P
P3
Π3
B2
A1
B
B3
P1
B1
Π1
A2
A3
|A3B3|=|AB|
β
P2
α
B3
B2
P3
A1
P1
B1
c

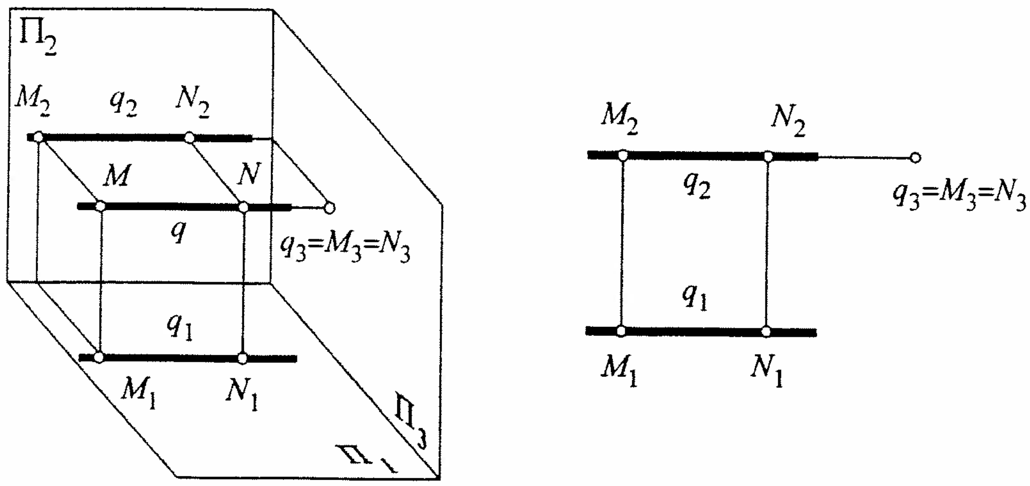
Fig. 12


Π2
A2
g2
A
B2
g
B
g1=A1=B1
Π1
A2
g2
B2
g1=A1=B1

a

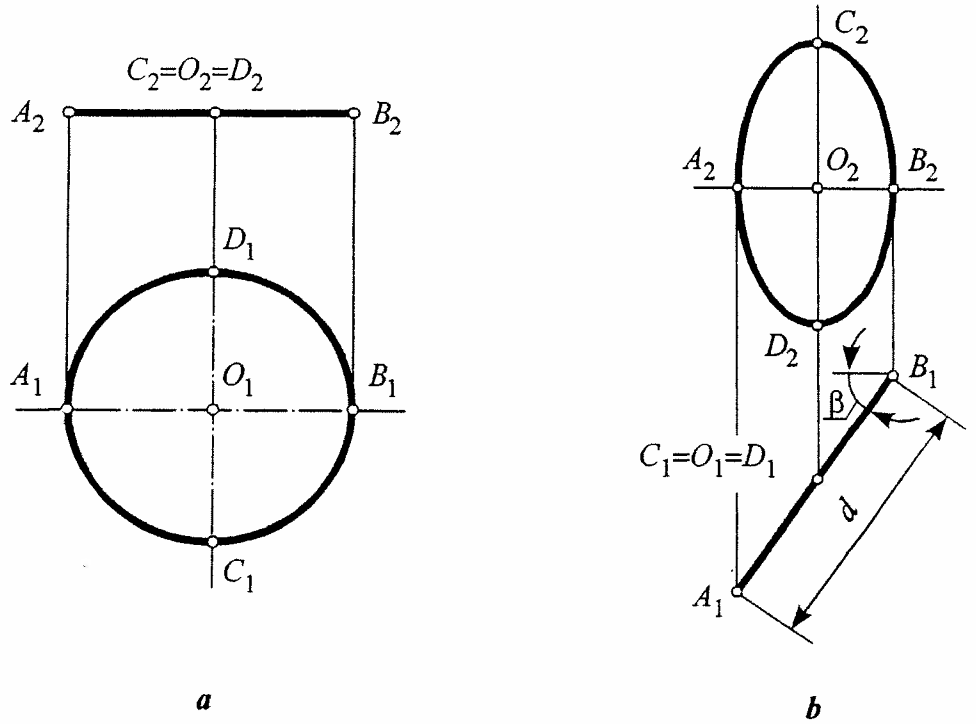

Π2
i2=D2=C2
C
i
D
C1
i1
D1
Π1
i2=D2=C2
C1
i1
D1

b


Π2
M2
q2
N2
M
N
q
q3=M3=N3
q1
M1
N1
Π3
Π1
M2
N2
q2
q3=M3=N3
q1
M1
N1

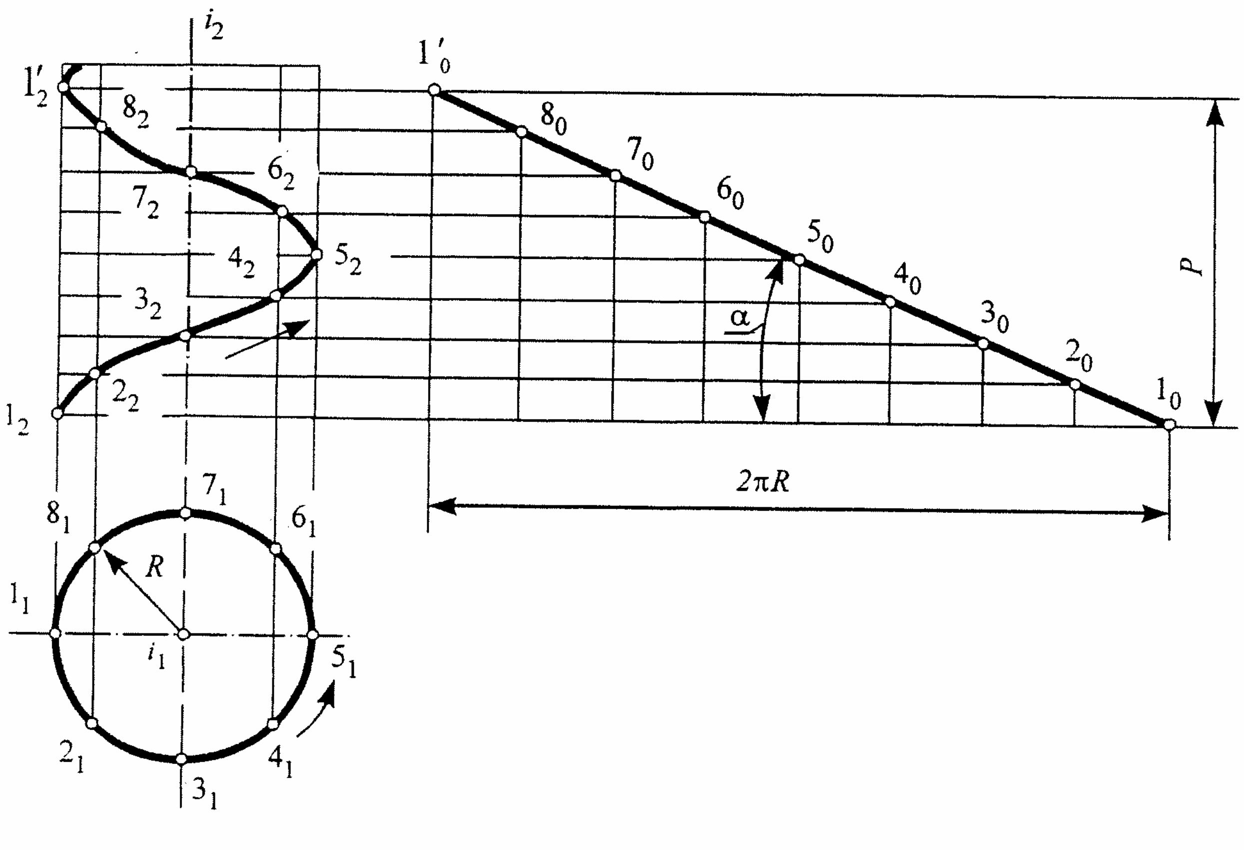
c

Fig. 13



C2=O2=D2
A2
B2
D1
A1
O1
B1
C1
a
C2
A2
O2
B2
D2
B1
β
C1=O1=D1
d
A1
b

Fig. 14



i2
1′2
82
62
72
42
52
32
22
12
1′0
80
70
60
50
40
30
20
10
α
P
2πR
71
81
61
R
11
i1
51
21
31
41

Fig. 15

A straight line is a development of a cylindrical helical line. The angle α is called a helix angle of the helical line. The operation of screw pairs (screw – nut) is based on the displacement property of the helical line meaning that each line-segment can displace along it without changing its form.

2.3. Surfaces

Surfaces limiting various technical forms are divided into plane, polyhedron and curved ones. Machine parts are very often composed of the combination of simplest geometrical objects such as prisms, pyramids, cylinders, cones, spheres and tores.

2.3.1. Plane. Task on the Drawing. A Point and a Line Belonging to a Plane

The plane is the simplest surface. Depending on the position of the plane relative to projection planes there are some types of planes:

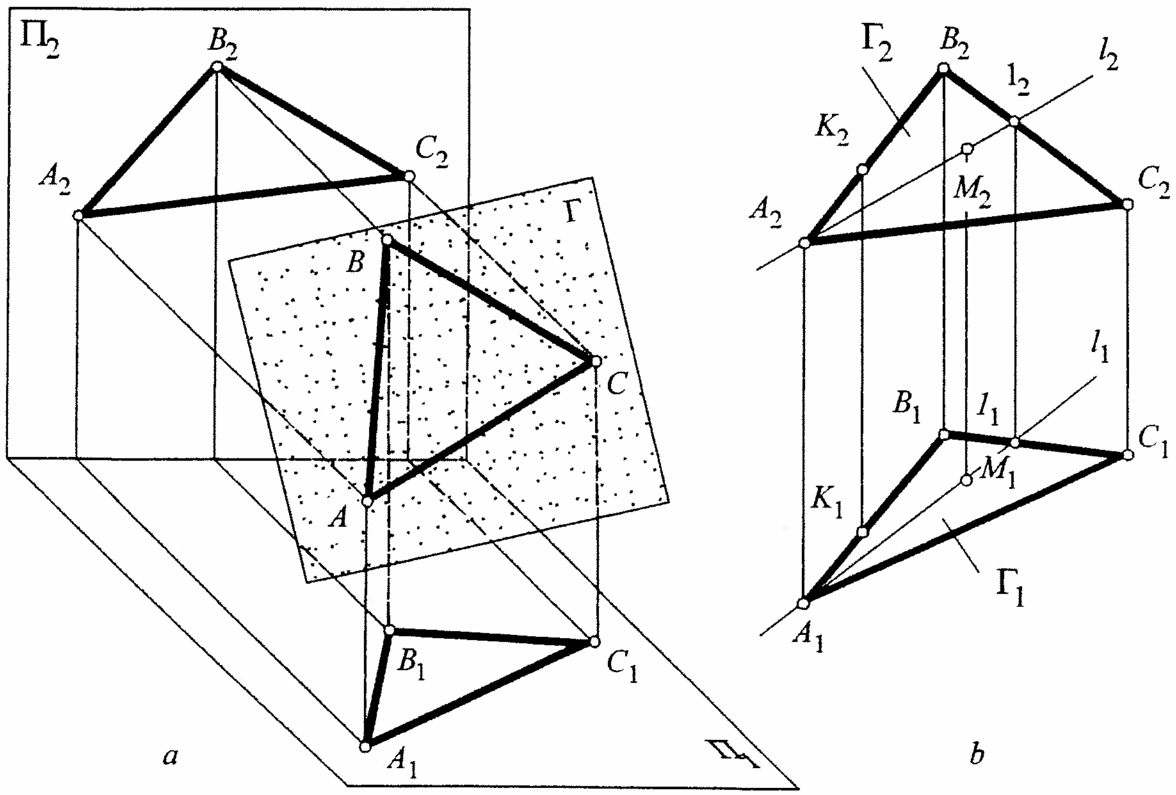


Fig. 16

1) the plane of a general position, which is non-perpendicular and non-parallel to the projection planes (Fig. 16);

2) the projecting plane, which is perpendicular to one of the projection planes (Fig. 17);

3) the level plane, which is parallel to one of the projection planes (Fig. 18).

The plane is given on the drawing by projections of geometrical objects determinating its position in space:

— of three points not lying on one line (Fig. 19, *a*);

— of a line and a point not lying on this line (Fig. 19, *b*);

— of two intersecting lines (Fig. 19, *c*);

— of two parallel lines (Fig. 19, *d*);

— of a plane object (Fig. 19, *e*).

Projections of both the point and the line belonging to the given plane are constructed by using the following axioms:

1) only one line passes through any two points;

2) the line passing through any two points of the plane belongs to this plane.

In Fig. 16, *b* the point *K* belongs to the plane $\Gamma(ABC)$, since it belongs to one of the lines [*AB*] representing the plane, where

$$K_2 \in A_2B_2 \wedge K_1 \in A_1B_1 .$$

To draw the line *l* belonging to the plane $\Gamma(ABC)$ it is required to draw it through any two points belonging to this plane, e.g. points *A* and *1*. The point *M*, belonging to the plane $\Gamma(ABC)$, can be obtained on the line drawn:

$$M \in \Gamma(ABC) \Leftrightarrow M \subset l \wedge l \subset \Gamma(ABC) .$$

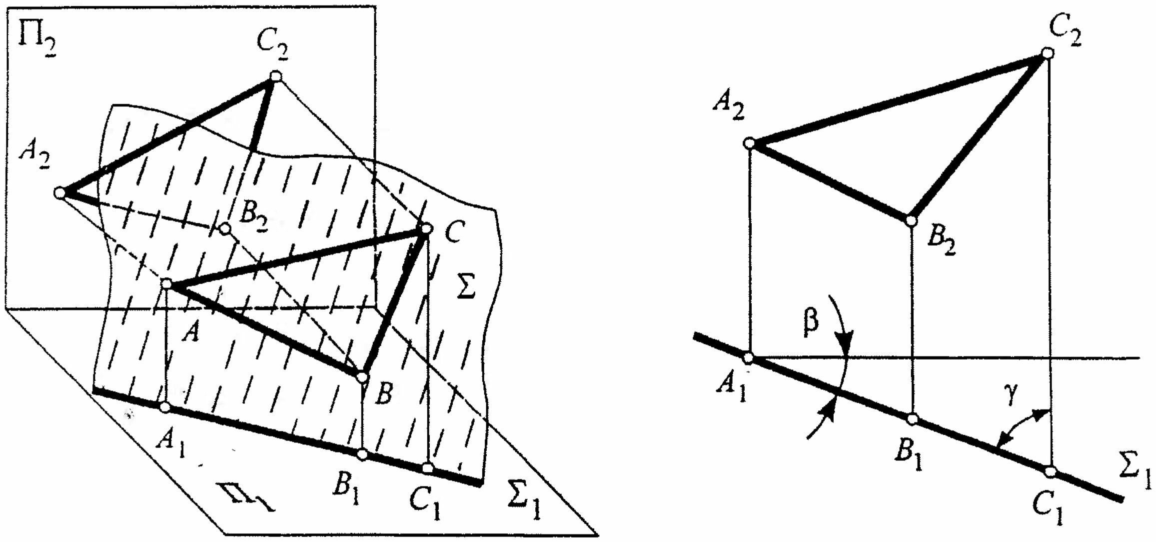
In Fig. 17 and Fig. 18 drawings of planes of particular position are given, i.e. of projecting and level planes.

Depending on to what projection plane the projecting plane is perpendicular, it is called a horizontal projecting (Fig. 17, *a*), a frontal-projecting (Fig. 17, *b*) and a profile-projecting plane (Fig. 17, *c*).

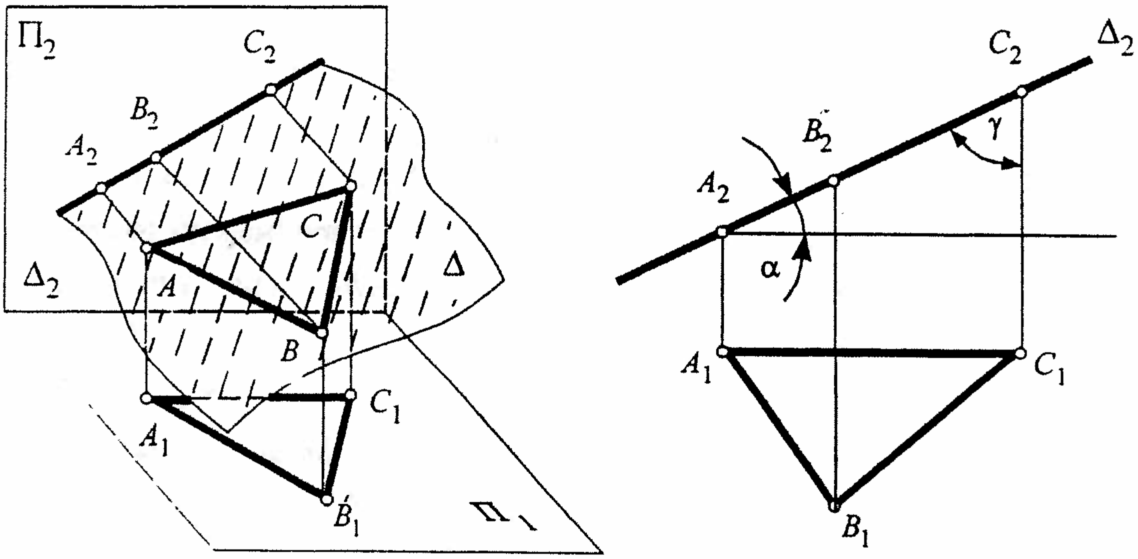
On the complicated drawing projections of geometrical objects representing the projecting plane (and belonging to it) will:

— degenerate into a straight line on the projection plane (Π_1 in Fig. 17, *a*; Π_2 in Fig. 17, *b*; Π_3 in Fig. 17, *c*), perpendicular to the projecting plane;

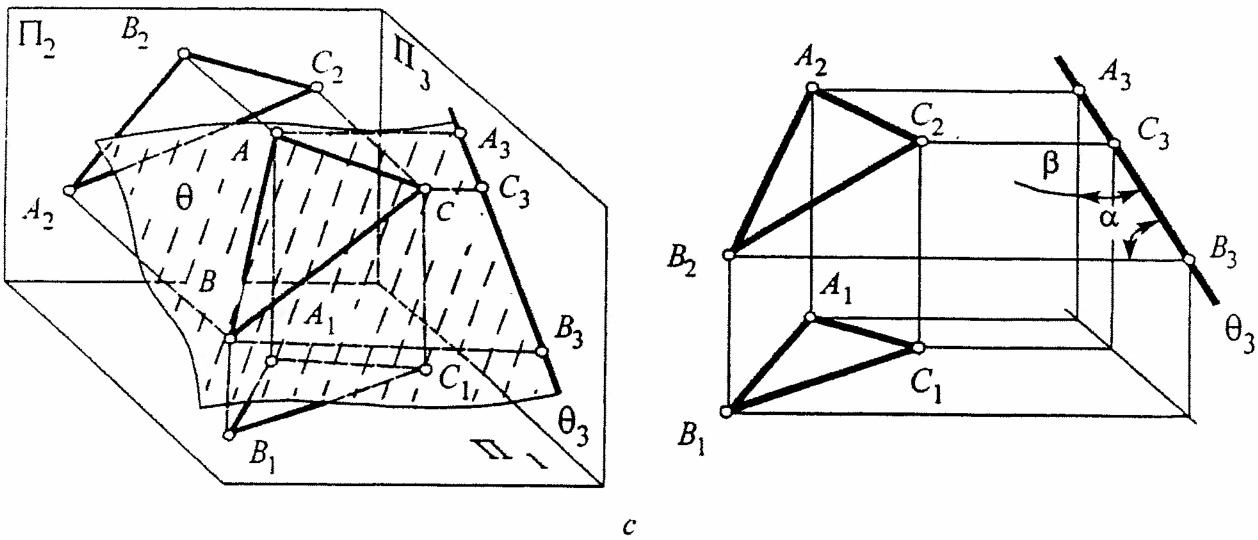
— represent a locus of points coinciding with the locus of plane points on other projection planes (Fig. 17).


Π_2
C_2
A_2
B_2
C
Σ
A
B
A_1
Π_1
B_1
C_1
Σ_1
β
γ

a

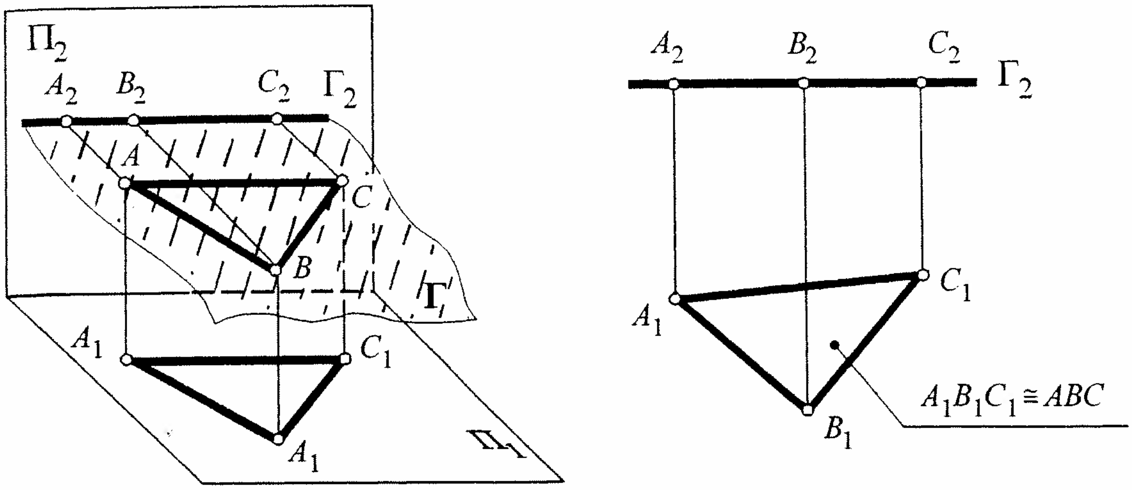

Π_2
C_2
B_2
A_2
C
Δ
Δ_2
A
B
C_1
A_1
B_1
Π_1
γ
α

b


Π_2
B_2
C_2
Π_3
A
A_3
θ
C_3
C
A_2
B
A_1
B_3
C_1
θ_3
B_1
Π_1
β
α

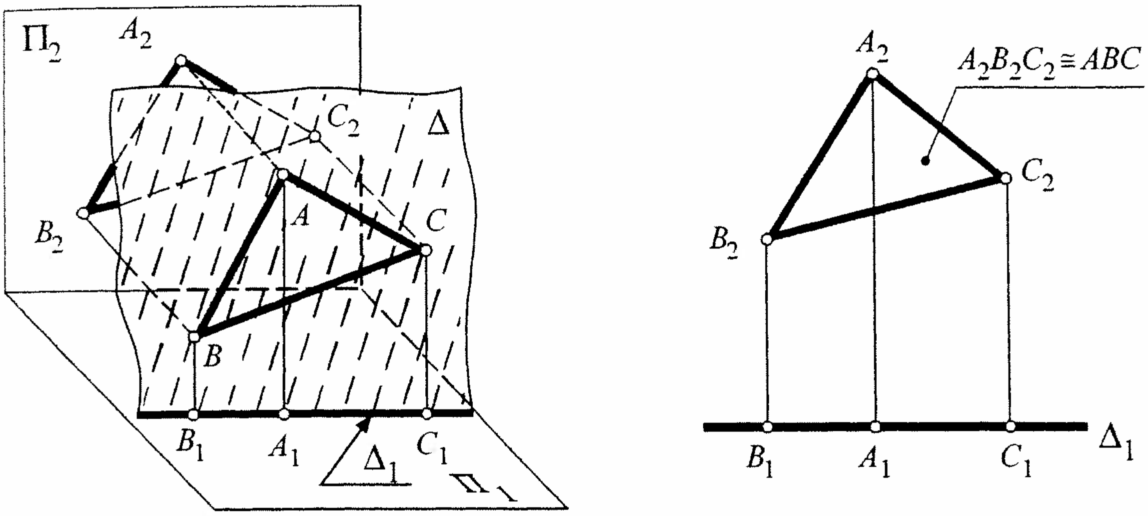
c

Fig. 17



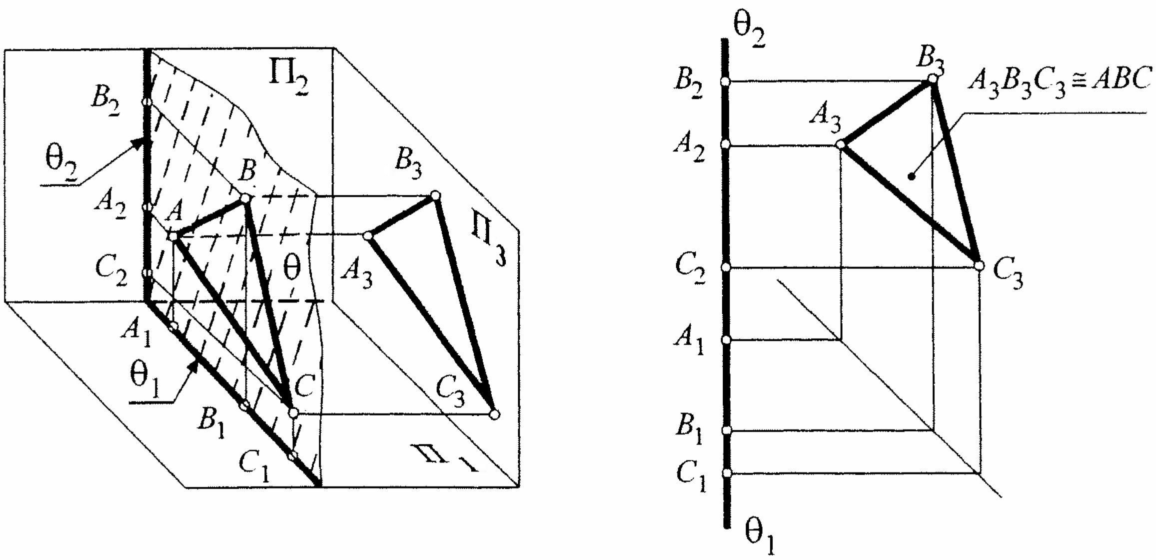
Π2
A2
B2
C2
Γ2
A
C
B
Γ
A1
C1
A1
Π1
A1B1C1 ≅ ABC
B1

a



Π2
A2
C2
Δ
A
C
B2
B
B1
A1
C1
Δ1
Π1
A2B2C2 ≅ ABC

b



Π2
B2
θ2
A2
B
A
B3
Π3
C2
θ
A3
A1
θ1
C
C3
B1
C1
Π1
A3B3C3 ≅ ABC

c

Fig. 18

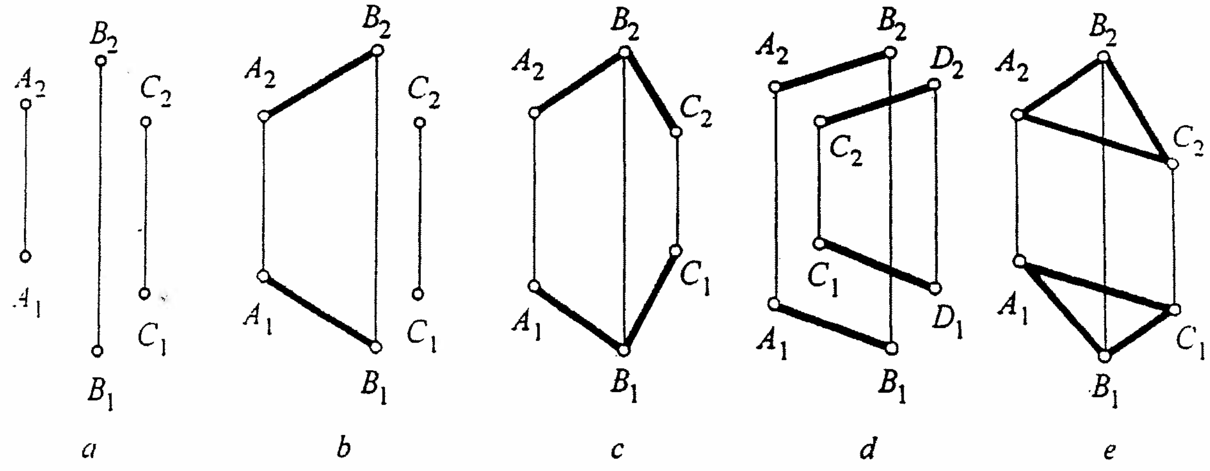


Fig. 19

Depending on to what projection plane the level plane is parallel, it is classified into a **horizontal level plane** (рис. 18, *a*), a **frontal level plane** (Fig. 18, *b*) and a **profile level plane** (Fig. 18, *c*).

On the complicated drawing geometrical projections representing the level plane (and lying on it) will:

— be congruent to their true value on the projection plane which is parallel to the level plane (e.g. in Fig.18, *a* $\Gamma \parallel \Pi_1 \wedge \Delta ABC \subset \Gamma \Rightarrow \Delta A_1B_1C_1 \cong \Delta ABC$);

— degenerate into a straight line on the projection planes which are perpendicular to the level plane (Fig. 18).

If an object occupies a profile position (Fig. 18, *c*), the drawing containing its horizontal and frontal projections will be unreversible. In order to represent the shape of the given object it is required to have a profile projection or to construct this projection, if horizontal and frontal projections of object vertices are marked.

2.3.2. Face Surfaces. Polyhedrons

Displacing the straightforward generating *l* along the broken guide *m* forms face surfaces. If one point *S* of the generating line is fixed, the surface of a pyramid is created (Fig. 20, *a*). If the generating line during this displacement is parallel to the given distance *S*, then the surface of a prism is created (Fig. 20, *b*).

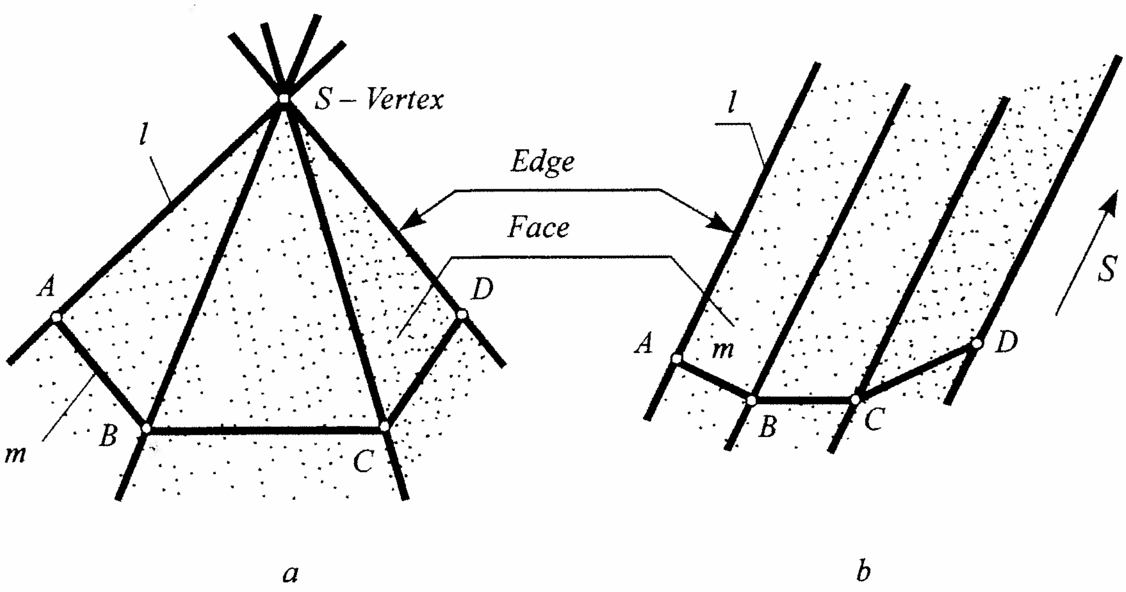
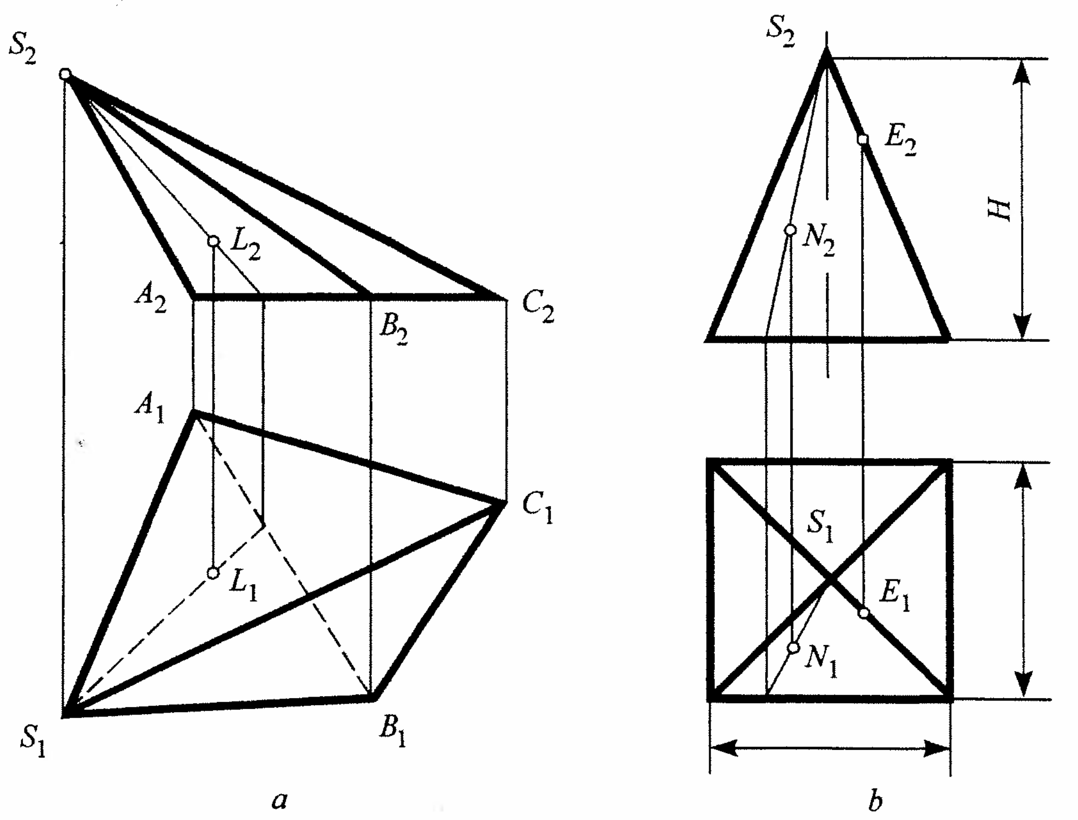


Fig. 20

Polyhedrons are closed geometrical objects bounded by flat polygons. To construct projections of polyhedrons means to draw their edges and vertices, i.e. lines and points. Drawings of the simplest polyhedrons, i.e. of pyramids and prisms are shown in Fig. 21 and Fig. 22. The number of polyhedron, projections must ensure the reversibility of the drawing. A drawing is called a reverse one, if having one point projection of plane another projection can be constructed. In a general case a two-projection drawing of the polyhedron containing horizontal and frontal projections is considered to be a reverse one, unless it has coinciding projections of edges and none of the edges is the profile line (Fig. 21 and 22). If these conditions are not met, then to make the drawing reverse it is required to construct the third projection of the polyhedron (Fig. 23, *a*) or to define all its vertices. For a cube and a rectangular parallepipid a three-projection reverse drawing is used (Fig. 23, *b*).

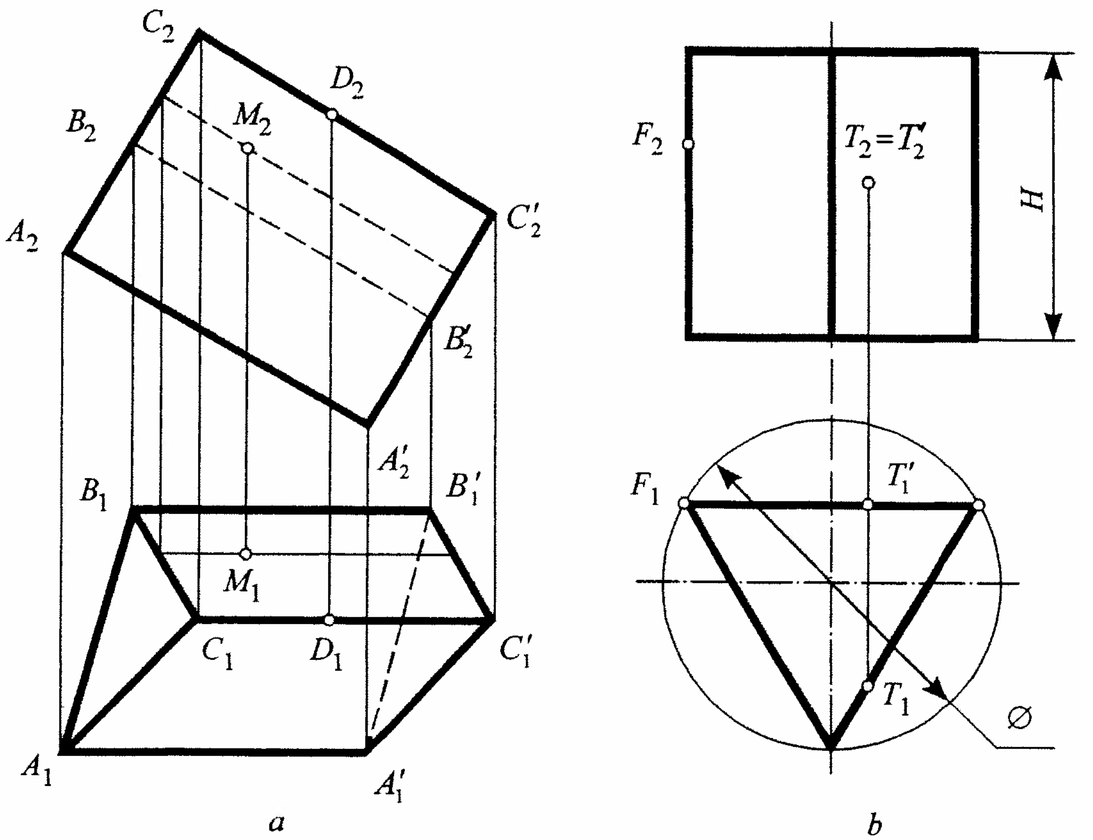
Closed broken lines $S_1A_1C_1B_1$ (Fig. 21, *a*) and $A_1B_1B_1'C_1'A_1'$ (Fig. 22, *a*) are the outlines of the horizontal projection of polyhedrons, while closed broken lines $S_2A_2C_2$ (Fig. 21, *a*) and $A_2C_2C_2'A_2'$ (Fig. 22, *a*) are the outlines of the frontal projection of polyhedrons. The projection outline is always visible.

In Fig. 22, *b* the horizontal projection of the prism coincides with the projection of its base since edges are horizontal-projecting lines, whereas prism bases are congruent objects of level planes. Faces are horizontal-projecting planes, therefore horizontal projections of points *F*, *T* and *T′* coincide with edge projections. The base of a regular four-angle pyramid in Fig. 21, *b* is a horizontal level plane.



S2
L2
A2
B2
C2
A1
C1
L1
S1
B1
a
S2
E2
N2
H
S1
E1
N1
b

Fig. 21



C2
D2
B2
M2
A2
C'2
B'2
A'2
B1
B'1
M1
C1
D1
C'1
A1
A'1
a
F2
T2=T'2
H
F1
T'1
T1
⌀
b

Fig. 22

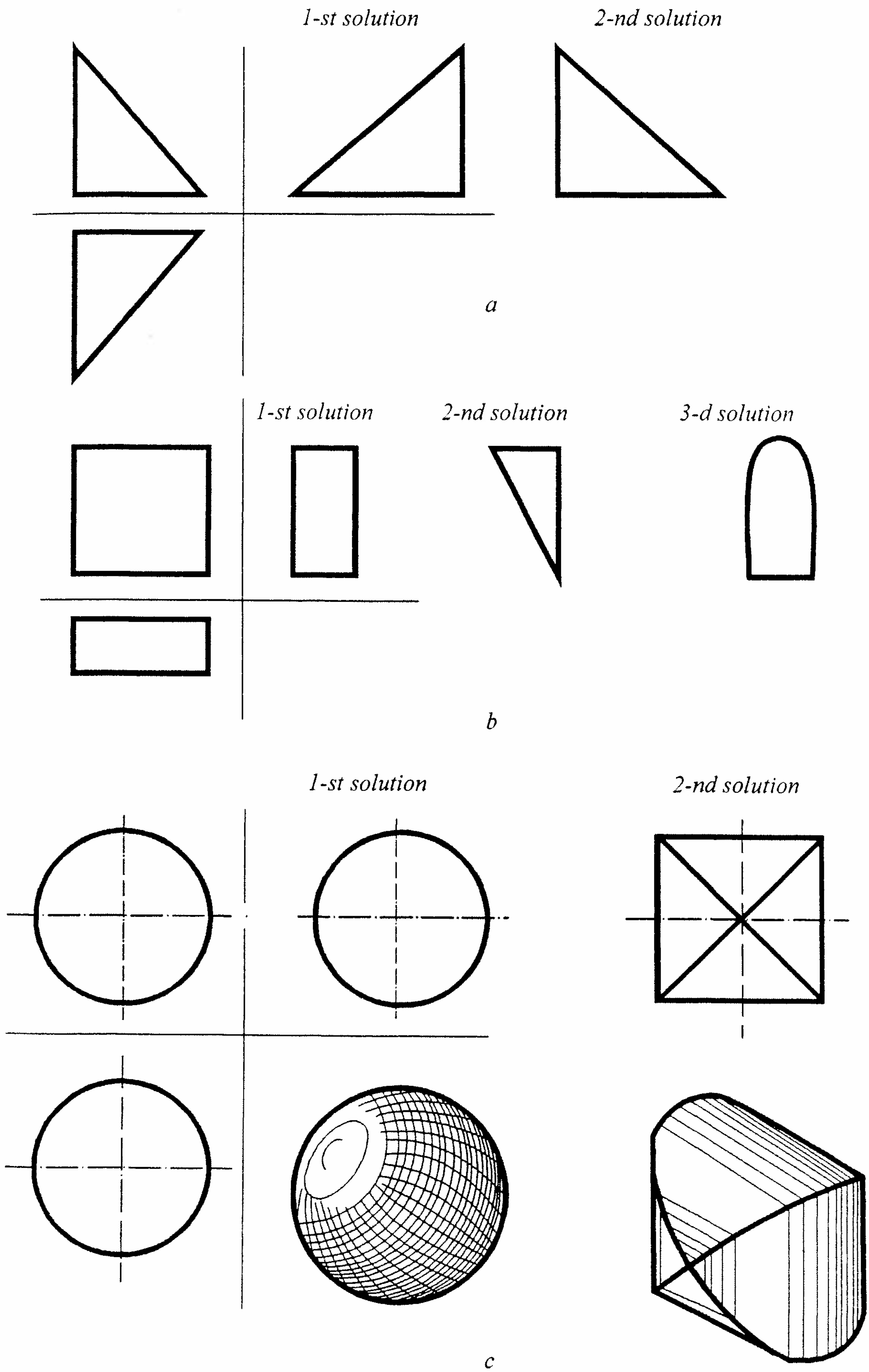

1-st solution
2-nd solution
a
1-st solution
2-nd solution
3-d solution
b
1-st solution
2-nd solution
c

Fig. 23

The frontal and back faces are profile-projecting planes (they pass through frontal and back base edges which are profile-projecting lines), whereas two side faces of the pyramid are frontal-projecting planes. The construction of points lying on the face or the edge of the prizm or the pyramid is similar to the construction of points lying on the straight line or the plane, as shown in 2.2.1. and 2.3.1. Points and lines lying on visible face projections on the drawing are also visible (points L_2 in Fig. 21, *a*; N_1 and N_2 in Fig. 21, *b*; M_1 in Fig. 22, *a*). If the projection of the face is invisible, then the respective projection of the point lying on this face is also invisible (the points L_1 in Fig. 21, *a*; M_2 in Fig. 22, *a*; T_2' in Fig. 22, *b*).

To define the shape of a regular pyramid and a right-angle prizm on the drawing it is required to give only the height and the sizes of the base.

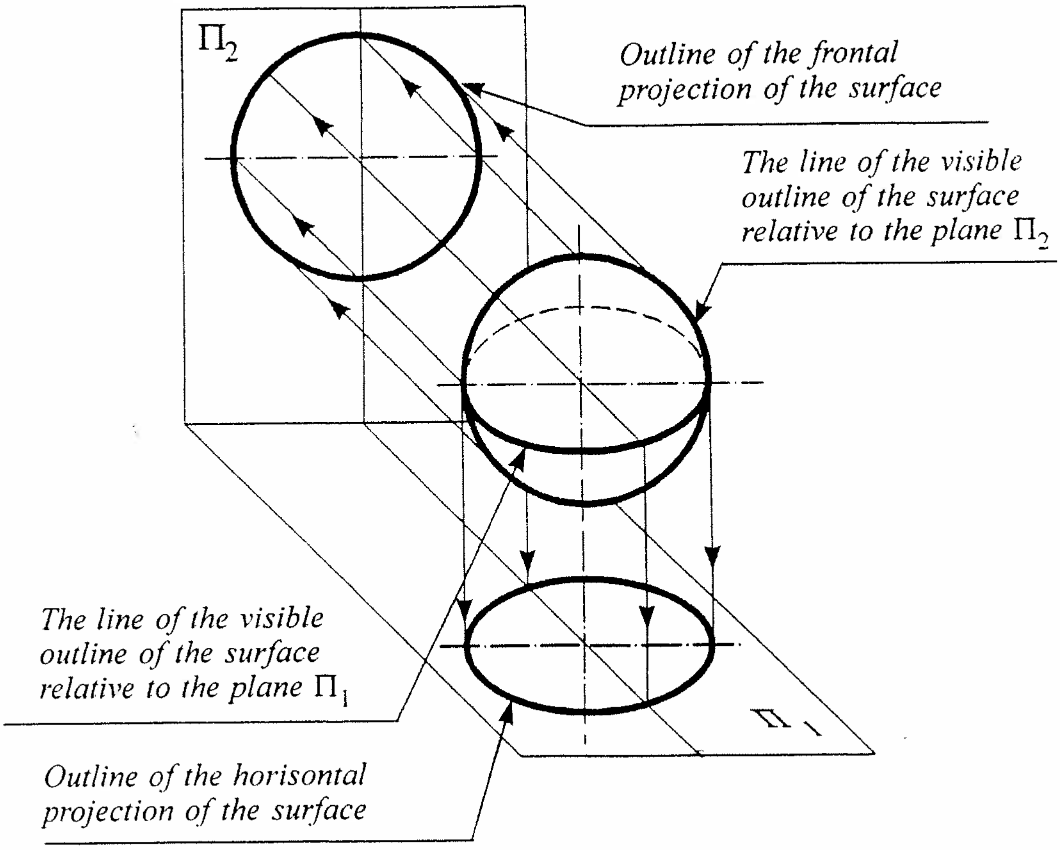
2.3.3. Rotation Surfaces

The rotation surface is constructed by rotating any line (generating) around the fixed axis. Depending on the type of the generating line (a straight line or a curve) rotation surfaces may be linear (a cylinder, a cone) and nonlinear (a sphere, a tore).

Each point of the generating line describes a circle, its plane being perpendicular to the axis of rotation (Fig. 24). These circles are called p a r a l l e l . Hence, the planes perpendicular to the axis intersect the rotation surface along the parallels. Major and minor parallels are called the e q u a t o r and the g o r g e c i r c l e , respectively. Planes passing through the axis of the rotation surface are called axis planes, the lines along which they intersect the surface are called m e r i d i a n n e s . The frontal meridian is called the main meridian which defines the frontal outline of the rotation surface. The profile meridian defines the profile outline of the rotation surface. All the meridiannes of the rotation surface are congruent.

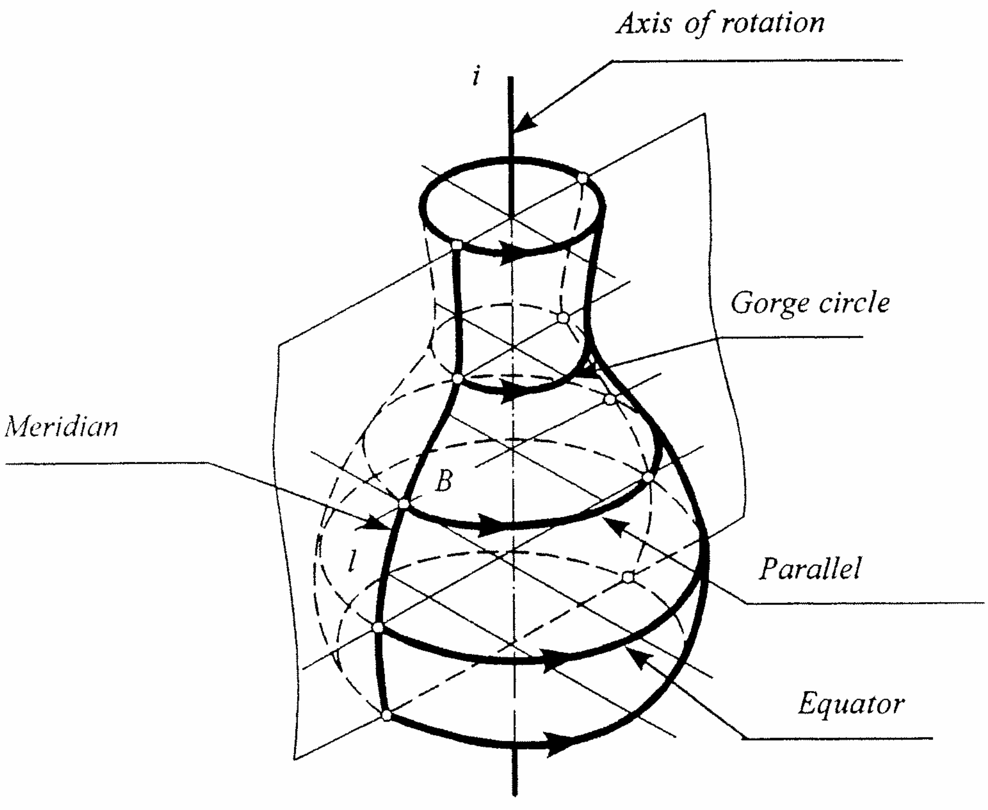
While projecting the surface on the projection plane the projecting rays intersect this surface at points which create a line called the line of a visible outline (Fig. 25).

The line of a visible outline of the surface divides it into two parts — the visible, turned to the viewer and the invisible one. For visibility the outlines of projections which are projections of corresponding lines of the visible outline are constructed on the surface drawing.



Π_2
Outline of the frontal projection of the surface
The line of the visible outline of the surface relative to the plane Π_2
The line of the visible outline of the surface relative to the plane Π_1
Π_1
Outline of the horisontal projection of the surface

Fig. 24



Axis of rotation
i
Gorge circle
Meridian
B
l
Parallel
Equator

Fig. 25

Fig. 26, *a* shows a straight circular cylinder formed by revolving the straight line around the axis which is parallel to the generating line. Since the cylinder axis is a horizontal-projecting line, the surface becomes the projecting one relative to the plane Π_1, whereas the horizontal projection of the plane is a circle. The horizontal projection of any point lying on the cylinder surface (on the drawing points A, A' and B), belongs to this circle.

For the construction of point projections lying on non-projecting surfaces the following rule is employed: the point belongs to the surface if it belongs to any surface line.

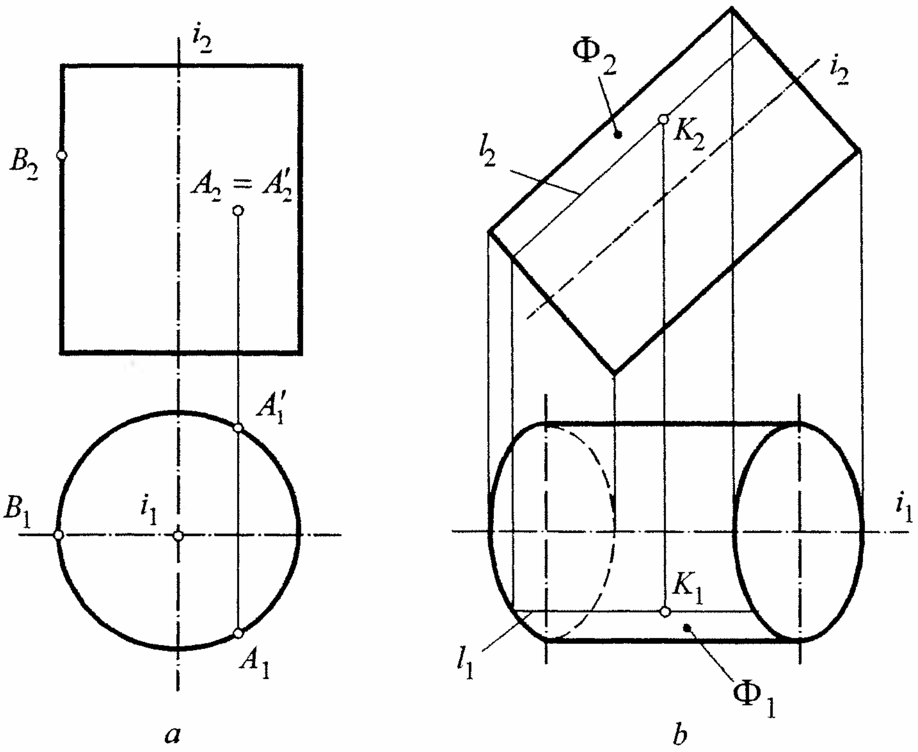


Fig. 26

In Fig. 26, *b* the cylinder axis is a frontal. The point K lying on the cylinder surface Φ, is drawn according to its belonging to the generating l:

$$K \in \Phi \Leftrightarrow K \in l \wedge l \subset \Phi \, .$$

The projections of points lying on the visible part of the cylinder surface on the drawing (the point A_2 in Fig. 26, *a*; points K_1 and K_2 in Fig. 26, *b*) are visible.

In Fig. 27 the right circular cone is shown formed by revolving the generating line around the axis intersecting this generating line. The cone axis is a horizontal-projecting line. The line of the visible outline of the cone surface relative

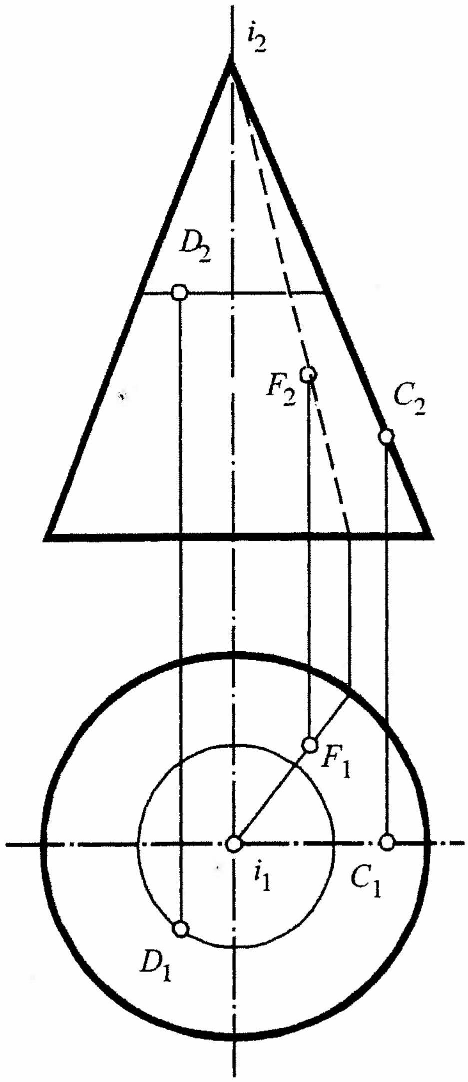


Fig. 27

to the plane Π_2 (its frontal projection is a triangle and the horizontal projection coincides with the horizontal axis) divides the conic surface into the visible surface situated closer to the viewer and the invisible one. The horizontal projection of the cone surface is visible. Point projections given on the cone surface are constructed taking into account their belonging to generating lines and circles. In Fig. 27 horizontal projections C_1, D_1 and F_1 of points C, D and F are visible, frontal projections C_2 and D_2 are visible and the point F_2 is invisible.

A sphere is constructed by the circle revolving around one of its diameters. The outline of the sphere's frontal projection is the projection of the main meridian n_2; the outline of the sphere's horizontal projection is the projection m_1 of the equator m (Fig. 28). The visibility boundaries on the sphere relative to projection planes are corresponding lines of the visible outline (Fig. 25).

On the frontal projection plane the visible part is the frontal part of the sphere's surface, and on the horizontal projection the visible part is the top part of the sphere's surface (Fig. 28). Construction of point projections belonging to the sphere's surface is carried out on the drawing with the aid of parallels to which points belong. In Fig. 28 all projections of the point K are visible. The frontal and horizontal projections of the point M as well as horizontal projection of the point N are visible, whereas projections N_2, N_3 and M_3 of points M and N are invisible.

All the above mentioned surfaces of rotation, i.e. the cylinder, the cone and the sphere are considered to be surfaces of the second order.

The torus is constructed by revolving the circle around the axis lying on the circle plane not passing through its center. The rotation axis can cross

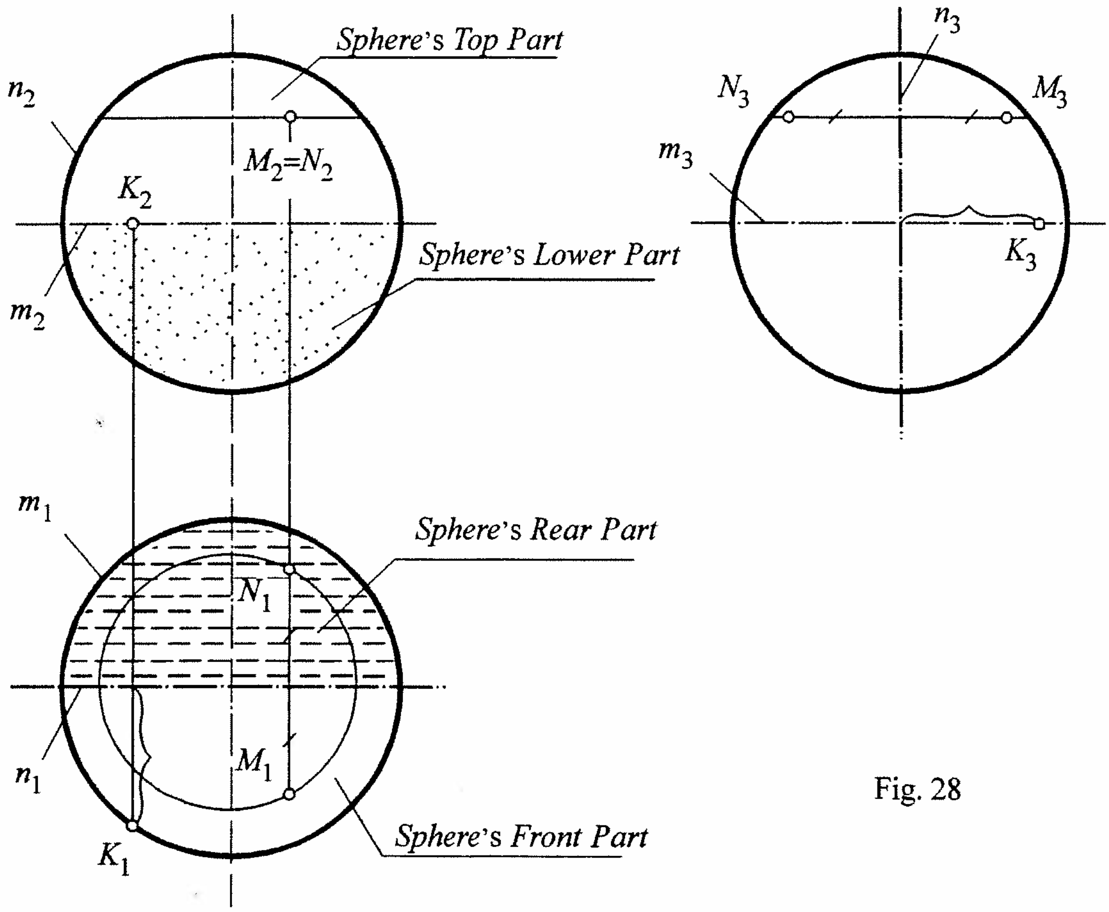

Sphere's Top Part
n_2
M_2=N_2
K_2
Sphere's Lower Part
m_2
n_3
N_3
M_3
m_3
K_3
m_1
Sphere's Rear Part
N_1
n_1
M_1
K_1
Sphere's Front Part

Fig. 28

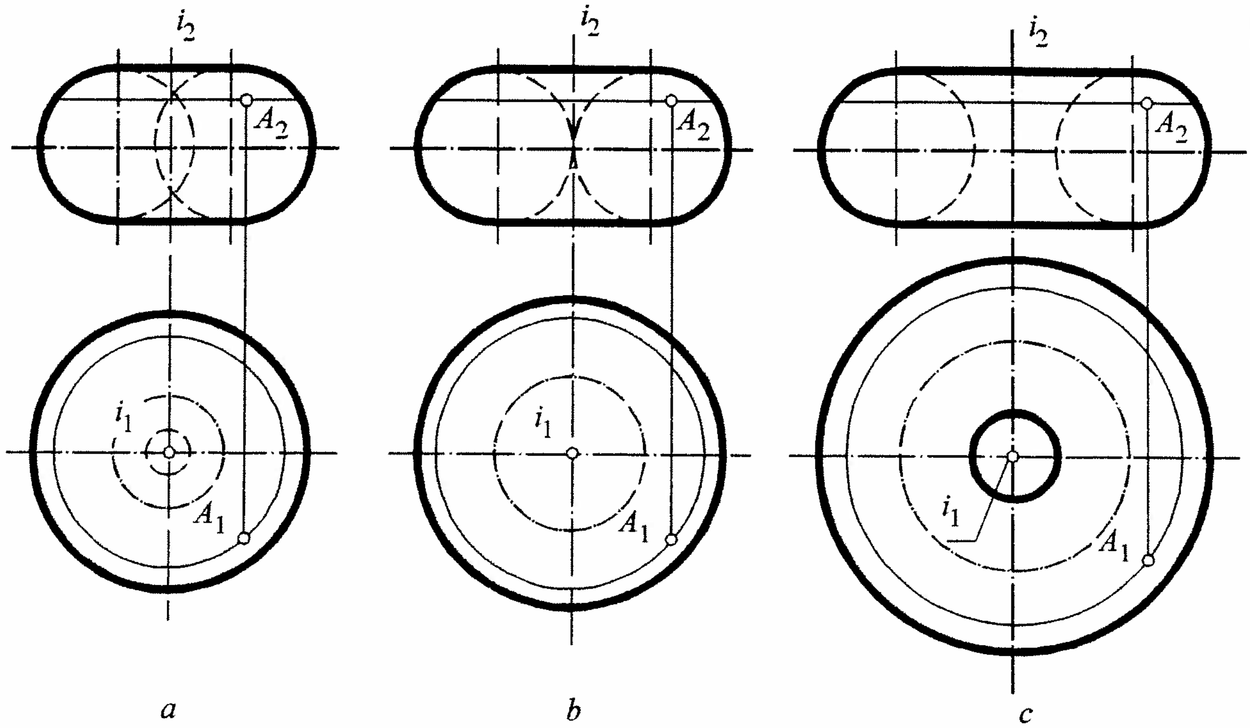

i_2
A_2
i_1
A_1
a
b
c

Fig. 29

the circle (Fig. 29, *a*), contact with it (Fig. 29, *b*) and lie outside the circle (Fig. 29, *c*). In the former two cases the torus is called closed, in the latter case it is called opened or circular. The torus is the surface of the fourth order.

The cylinder (Fig. 26), the cone (Fig. 27) and the torus (Fig. 29) will have the reverse drawing consisting of two projections (one of them is to lie on the plane perpendicular to the rotation axis). The shape of the sphere is not defined by these two projections. In Fig. 23, *c* two variants of the third projections are given (other solutions are also possible). A three-projection drawing of the sphere is a reverse one. Using conventional symbols and inscriptions stipulated by GOST 2.307-68, the number of projections can be reduced: the diameter sign ∅ tells that the object shown is the revolution body; signs *R* or ∅ may be preceded by the sign ○ indicating that the surface is a spherical one (Fig. 30). The right circular cylinder and the right circular cone are obtained by the diameter of the base and by the height (Fig. 30, *a*, *b*), whereas the sphere — by the diameter of the generating circle, and the torus — by the diameters of both the generating circle and the major parallel (Fig. 30, *c*, *d*).

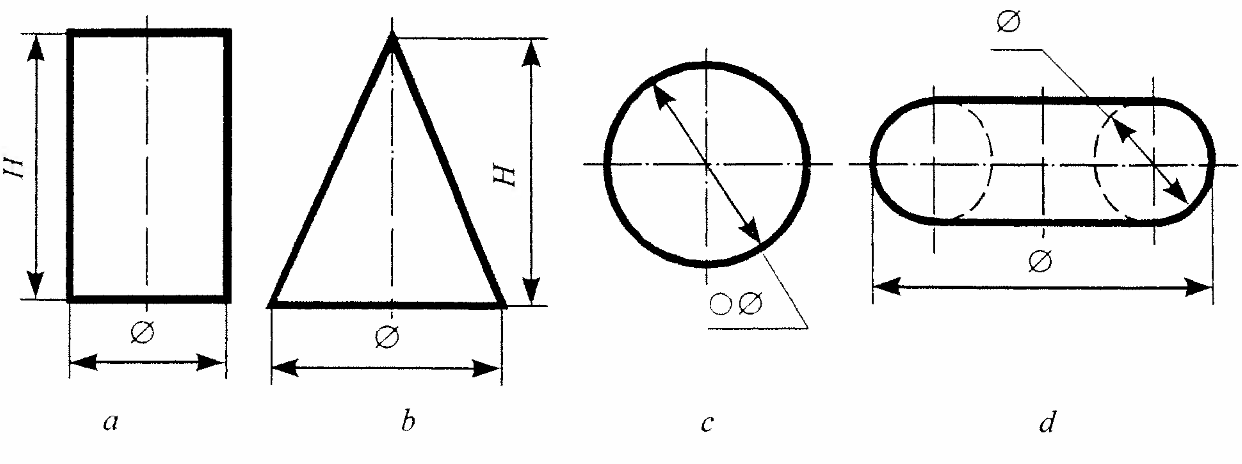


Fig. 30

P r o b l e m . Construct missing projections of points *A*, *B*, *C*, *D* lying on a combined surface. The given point projections A_1, D_1, B_2, C_2 are visible (Fig. 31, *a*).

Construction sequence:

1) determine what surfaces construct the given object;

2) choose a graphic simple line of the surface (a circle or a straight line) passing through the given point;

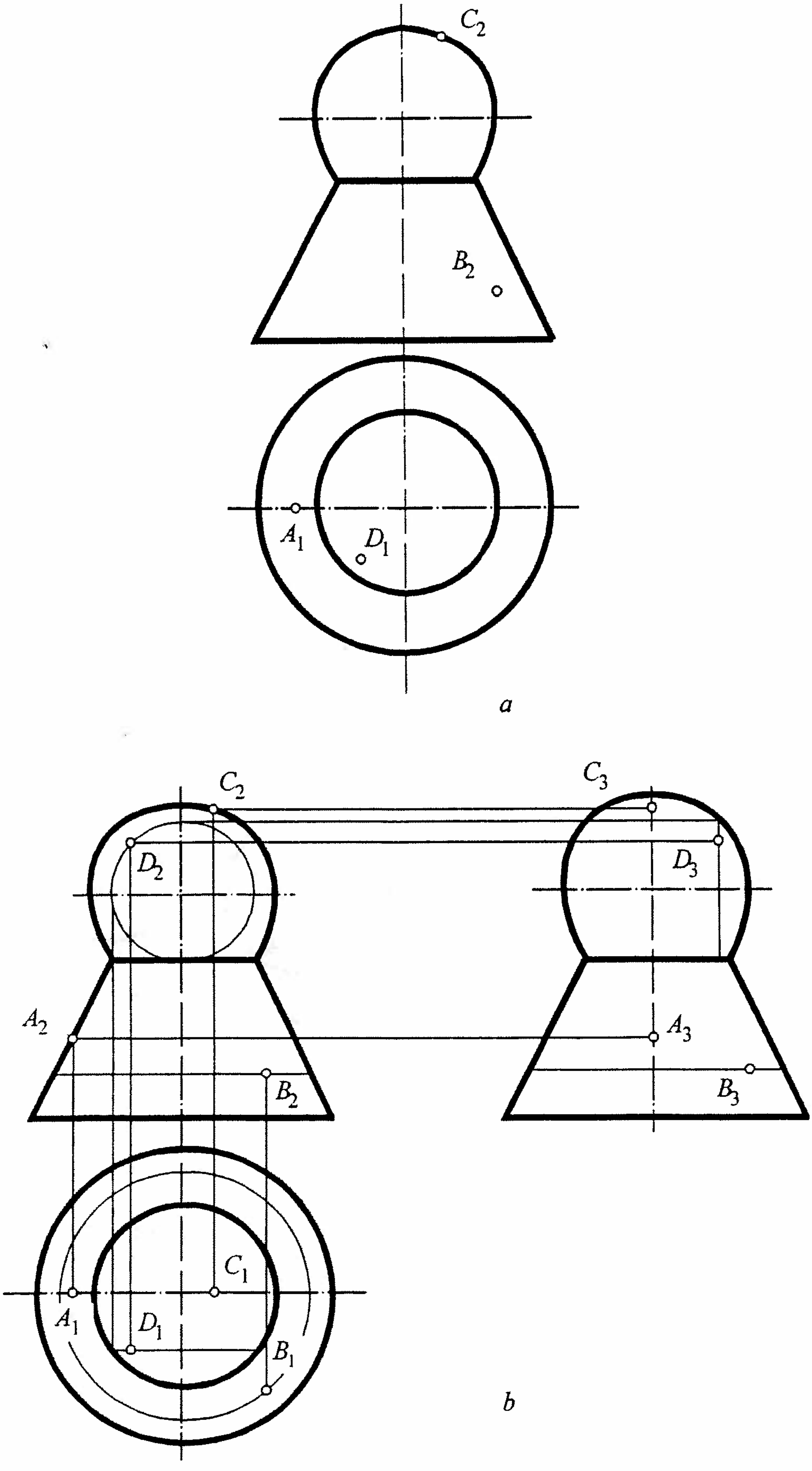


Fig. 31

3) construct projections of this line;

4) construct the required projections of the given point.

S o l u t i o n . The given combined surface is a combination of a truncated cone and a sphere's portion.

Points A and B lie on a conical surface. Construction of projections A_2 and A_3 of the point A is carried out on account of this point's belonging to the straight line of the cone generator. To draw points B_1 and B_3 it is required to draw the cone parallel through the point B.

Points C and D lie on the sphere's surface. Projections C_1 and C_3 are constructed on account of the point C belonging to the principal meridian of the sphere. To draw points D_2 and D_3 it is required to draw the sphere's parallel passing through the point D and parallel to the frontal plane of projections.

Constructions on the drawing (Fig. 31, *b*) are carried out according to the construction procedure (points 3 and 4).

§ 3. Determining the True Value of Plane Figures and Straight Lines by Means of the Projection Transformation Method

While drawing straight lines and plane objects it is necessary to locate them relative to the projection planes so that they could occupy a particular position. Drawings enable to see true sizes and shapes of objects drawn as well as of their elements. However, it is not always possible to locate an object in such a way that all its elements (straight lines, plane objects) would occupy a particular position. Therefore, some methods are required to move elements from a general position to a particular one. For this purpose methods of transforming projections are employed. In one of them the position of projection planes remains unchanged, new projections of objects being constructed by changing its position in space (revolution and coincidence methods). In another method an object of projecting remains unchanged while the position of the system of projection planes is changed (method of replacing projection planes).

3.1. Method of Replacing Projection Planes

In this method a geometrical element is projected without changing its position in space onto a new plane replacing any principal plane. The position of the additional projection plane is chosen corresponding to the problem set (parallel to the geometrical element). An additional plane must be by all means perpendicular to the unreplaced projection plane.

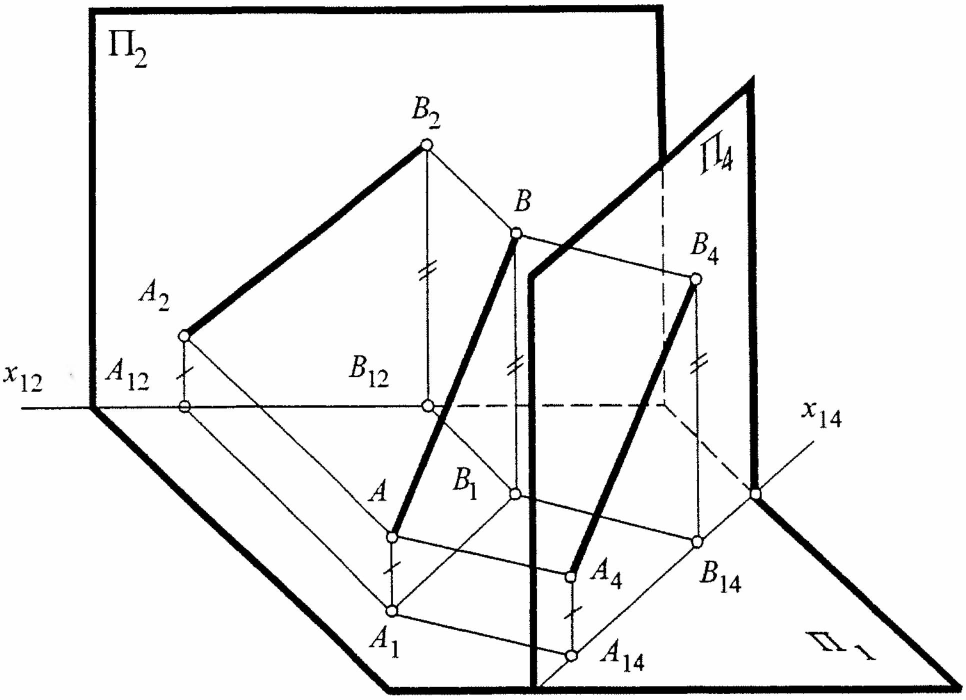
The method of replacing projection planes can be represented by the following example of determining the length of the line-segment $[AB]$ of the general position line (Fig. 32 *a*, *b*). It is known that the line-segment is projected on the projection plane as a congruent line-segment if it is parallel to it. Hence, to determine the segment length an additional plane should be located parallel to the line-segment $[AB]$. Replace the plane Π_2 with the plane Π_4 which is perpendicular to the plane Π_1 and parallel to the segment $[AB]$. The distance from the plane Π_4 to the segment $[AB]$ is taken at random (Fig, 32, *a*). Projection planes Π_1 and Π_4 form a new system of projection planes and intersect along the line x_{14} being a new projection axis which is in this case parallel to the horizontal projection of the segment, i.e. $x_{14} \parallel [A_1B_1]$. Here $|A_{12}A_2| = |A_{14}A_4|$ and $|B_{12}B_2| = |B_{14}B_4|$.

A_4B_4 is an orthogonal projection of the segment $[AB]$ on the plane Π_4,

$$[AB] \parallel \Pi_4 \Rightarrow |A_4B_4| = |AB| \cdot$$

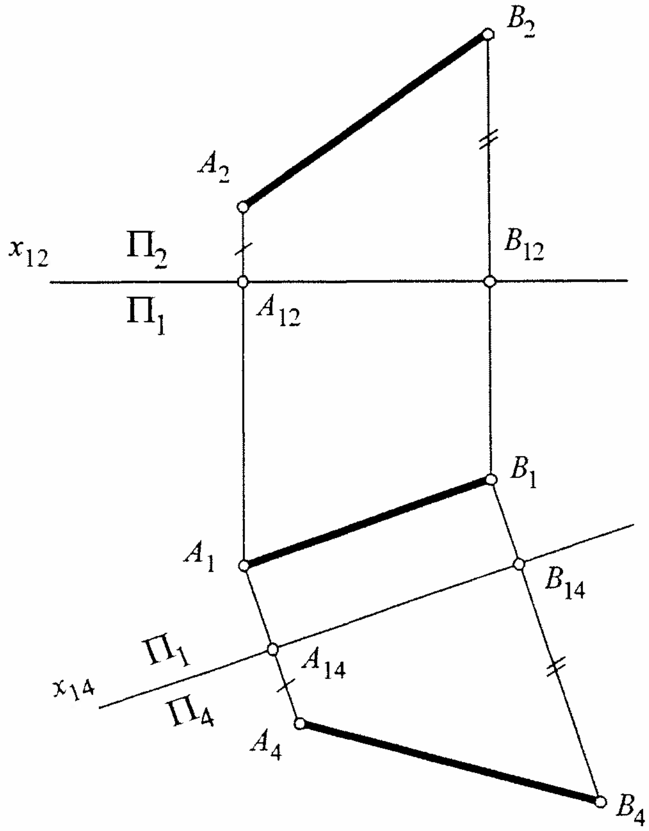
To obtain the projection A_4B_4 the segment $[AB]$ on the drawing proceed as follows. Draw a new axis x_{14} parallel to the horizontal projection of the segment $[A_1B_1]$ along a random distance from it. Through horizontal projections of end points of a line-segment (points A_1 and B_1) draw communication lines perpendicular to the axis x_{14}. On these lines from the axis x_{14} lay off segments $|A_{14}A_4| = |A_{12}A_2|$ and $|B_{14}B_4| = |B_{12}B_2|$. Joining points A_4 and B_4 by a straight line obtain the projection A_4B_4 of the segment $[AB]$ on the plane Π_4 (Fig. 32, *b*).

The problem of determining the true length of the segment of the general position line can be also solved with the aid of an additional projection plane taken instead of the plane Π_1 perpendicular to the plane Π_2 and parallel to the segment (Fig. 33 *a*, *b*).



Π_2
B_2
Π_4
B
B_4
A_2
x_{12}
A_{12}
B_{12}
x_{14}
A
B_1
A_4
B_{14}
A_1
A_{14}
Π_1

a



B_2
A_2
x_{12}
Π_2
B_{12}
Π_1
A_{12}
B_1
A_1
B_{14}
Π_1
A_{14}
x_{14}
Π_4
A_4
B_4

b

Fig. 32

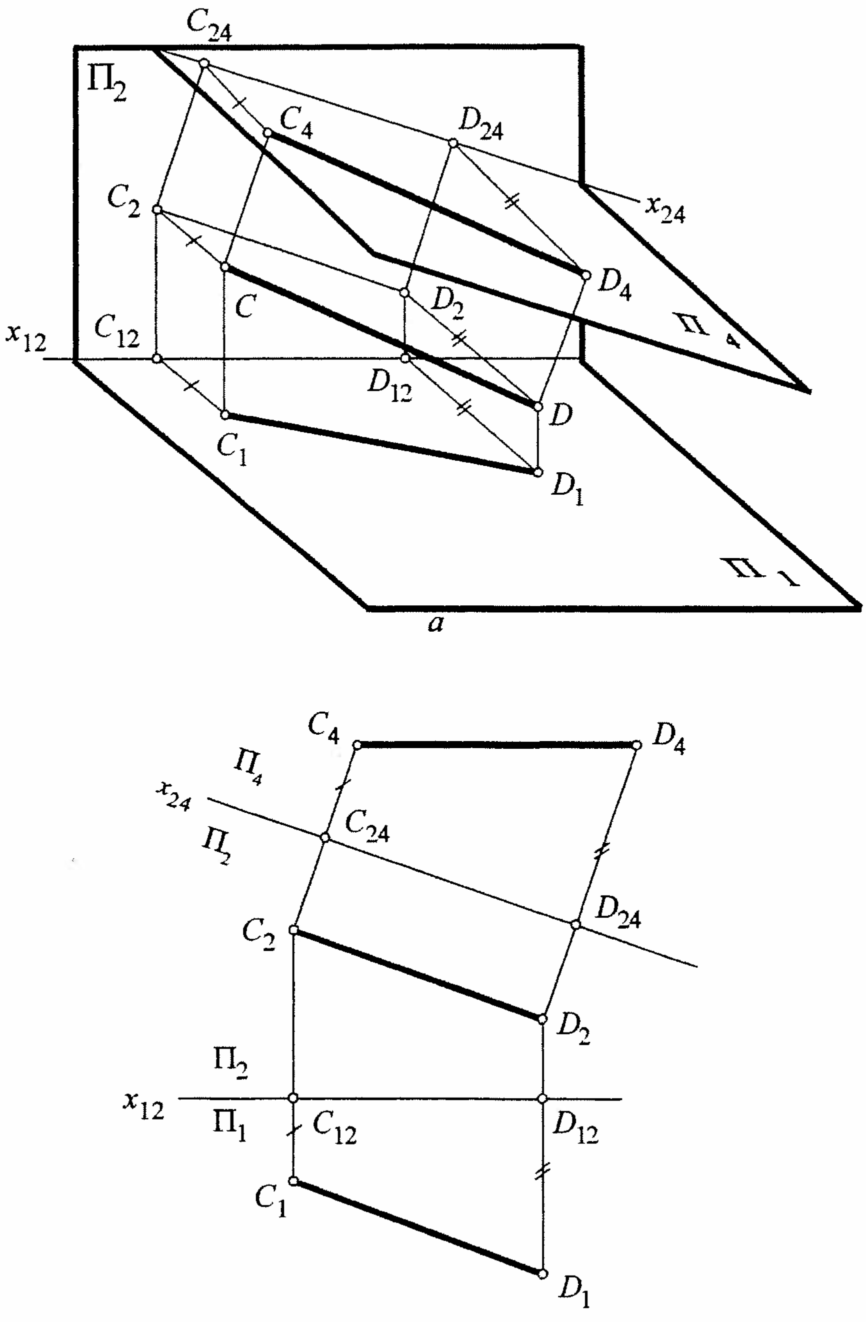


Fig. 33

The plane geometrical element is projected on the projection plane as a congruent object if it is located parallel to this plane. Hence, to determine the true value of the plane geometrical element a new additional plane must be located parallel to the object.

P r o b l e m. Determine the true value of the triangle $ABC \perp \Pi_2$ (Fig. 34).

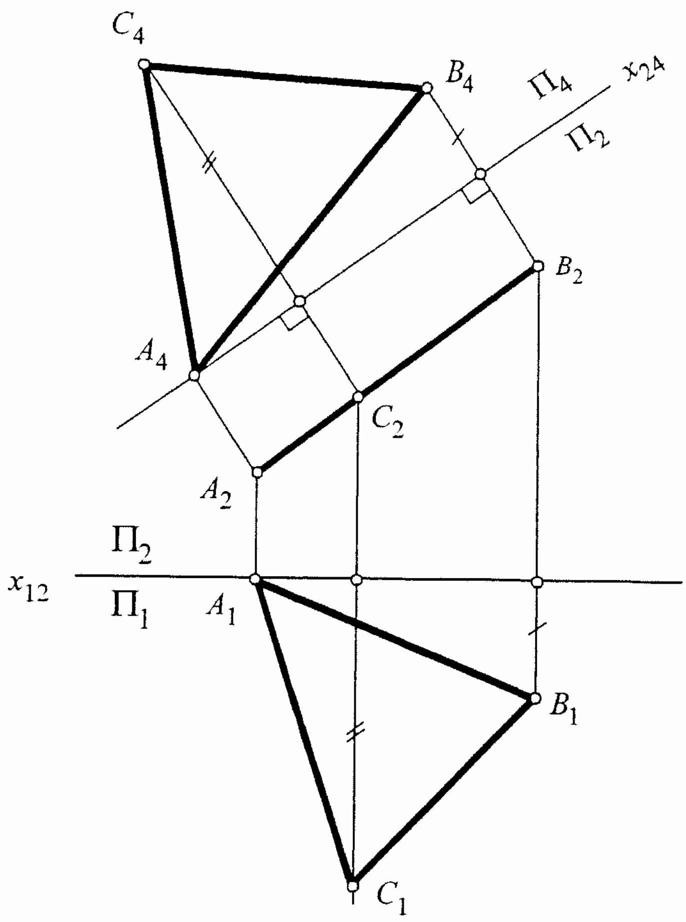


Fig. 34

Use an additional plane Π_4 locating it parallel to the triangle plane and perpendicular to the plane Π_2. Draw the new projection axis x_{24} parallel to the frontal projection of the triangle *ABC*. New projections A_4, B_4 and C_4 of the vertices *A*, *B* and *C* are obtained by laying off on communication lines from the axis x_{24} segments equal to the distance between points A_1, B_1, C_1 and the axis x_{12}.

The new projection $A_4B_4C_4$ obtained is the true value of the triangle *ABC*.

3.2. Method of Revolution

In this method a point, segment or plane object is revolved till the required position around a certain fixed line, i.e. the revolution axis. Any line can be taken as the revolution axis. To simplify the construction on the complicated drawing the projecting line is sometimes considered to be the revolution axis.

Let's consider the revolution of the point *A* around the horizontal projecting line i ($i \perp \Pi_1$) (Fig. 35, *a*). The plane $\Sigma \perp i$ on which the circle described by the point will be located becomes the horizontal plane of the level ($\Sigma \parallel \Pi_1$). Hence, the circle described by the point A in space will be projected on the plane Π_1 undistorted, whereas on the plane Π_2 – as a line-segment coinciding with Σ_2.

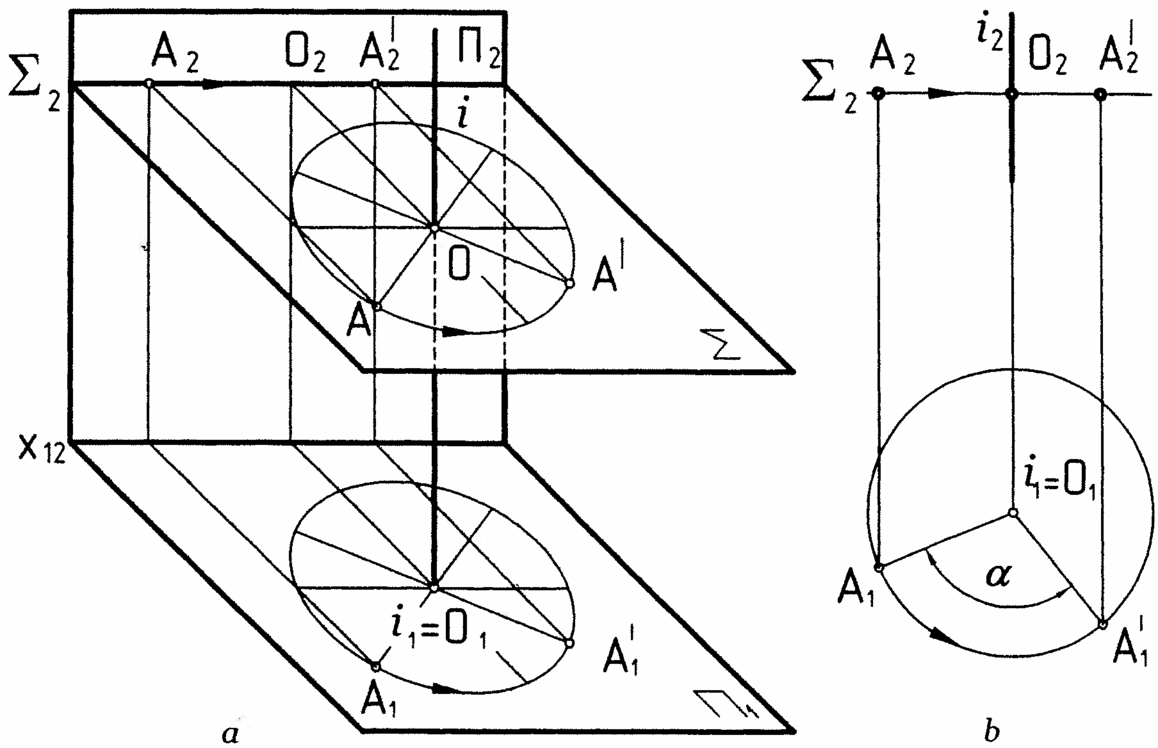


Fig.35

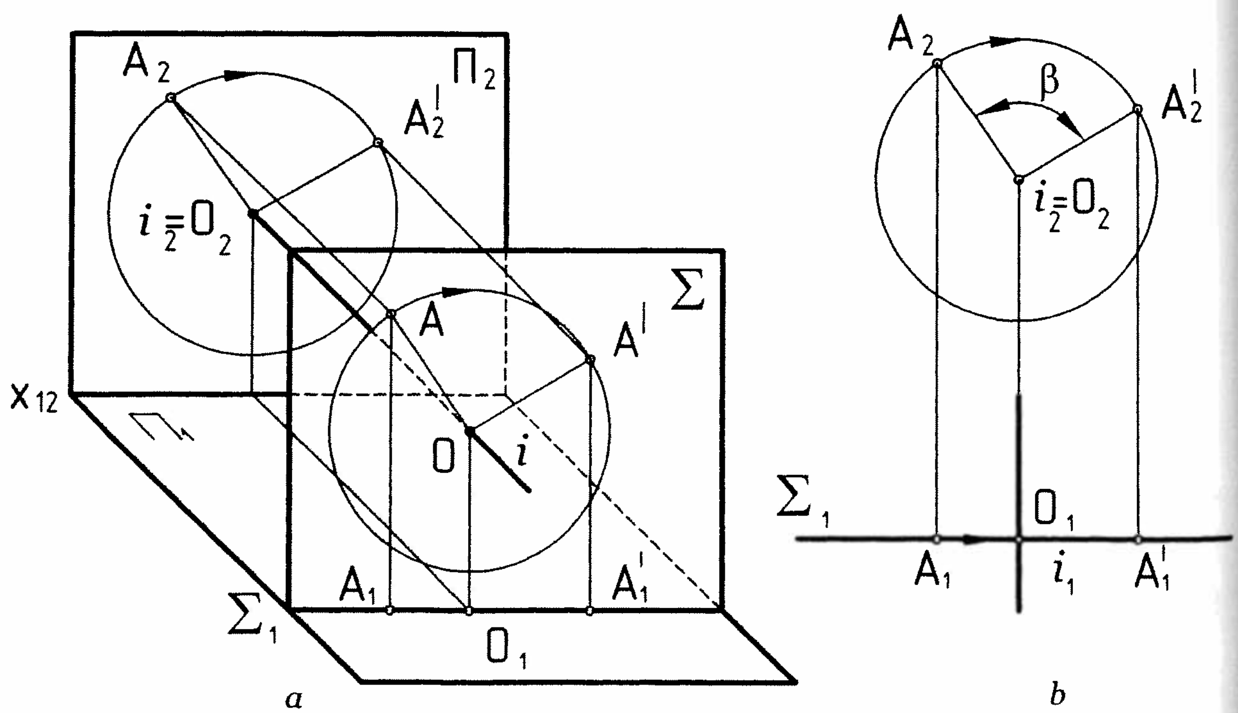


Fig.36

Thus, on the complicated drawing (Fig. 35, b) we have the following:

1) the horizontal projection A_2 of the point A is moving along the circle of the radius $|R| = |AO| = |A_1O_1|$;

2) the frontal projection A_2 of the point A is moving along a straight line perpendicular to communication lines (degenerated frontal projection Σ_2, of the plane $\Sigma \parallel \Pi_1$);

3) the revolution angle of the horizontal projection A_2 of the point A is equal to the revolution angle of the point in space.

If the point A revolves around the frontal projecting line i ($i \perp \Pi_2$) (Fig. 36, *a*), the plane Σ in which the circle described by the point is located becomes the frontal plane of the level ($\Sigma \parallel \Pi_2$). Hence, the circle described by the point A in space is projected on the plane Π_1 as a line-segment coinciding with Π_1, whereas it is projected on the plane Π_2 without distortion. Thus, on the complicated drawing (Fig. 36, *b*) we have the following:

1) the horizontal projection A_1 of the point A is moving along a straight line (degenerated horizontal projection Σ_1 of the plane $\Sigma \parallel \Pi_2$);

2) the frontal projection A_2 of the point A is moving along the circle of the radius $|R| = |AO| = |A_2O_2|$;

3) the revolution angle of the frontal projection A_2 is equal to the revolution angle of the point in space.

P r o b l e m. Determine the true value of the segment [*AB*] of general position (Fig. 37).

The true value of the segment |AB| can be defined by revolving it till the position parallel to the projection plane Π_2 or Π_1. Correspondingly, the revolution axis $i \perp \Pi_1$ or $i \perp \Pi_2$ can be used.

Draw the revolution axis i (i_1, i_2) $\perp \Pi_1$ through the point B (B_1, B_2) of the segment [*AB*] . While revolving the segment around the axis i, the point B remains fixed since it belongs to the axis, whereas the point A will revolve according to the rules given above. The revolution angle of the point A and its horizontal projection A_1' is defined in the following way: when the segment [*AB*] occupies the position [*AB*] $\parallel \Pi_2$, its horizontal projection $A_1'B_1$ will be perpendicular to communication lines. Further constructions are obvious from the drawing. $|A'B|$ — is the true value of the segment [*AB*].

P r o b l e m. Determine the true of the quadrangle $ABCD \perp \Pi_1$ (Fig. 38).

Since the plane of the quadrangle *ABCD* is a horizontal projecting plane, its true value may be obtained by revolving the quadrangle about the axis $i \perp \Pi_2$ till the position parallel to the plane Π_2. Draw the revolution axis *i* through one of its vertices, vertex *A*, for instance. Then the problem is reduced to revolving three quadrangle points *B, C* and *D.* Further constructions are based on the rules given above and are obvious from the drawing. The newly constructed projection $A_1B_1^{\prime}C_1^{\prime}D_1^{\prime}$ is the true value of the quadrangle *ABCD.*

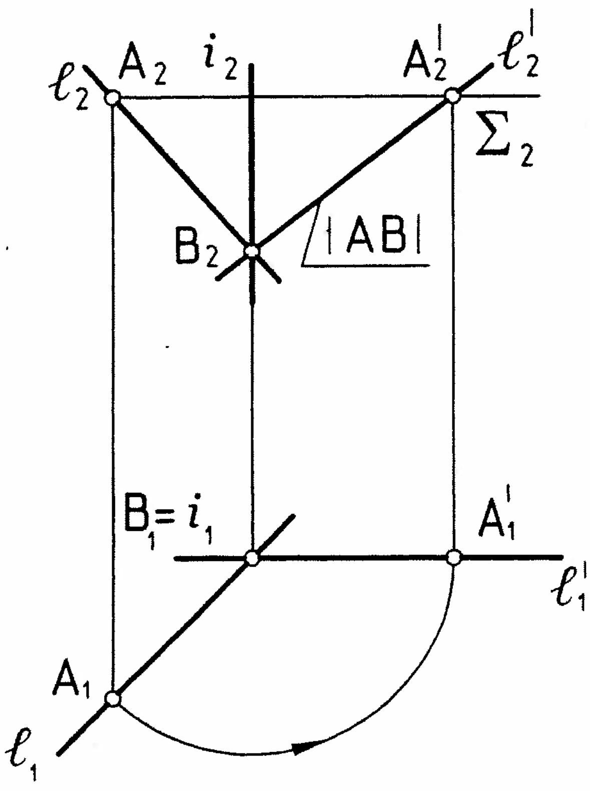


Fig.37

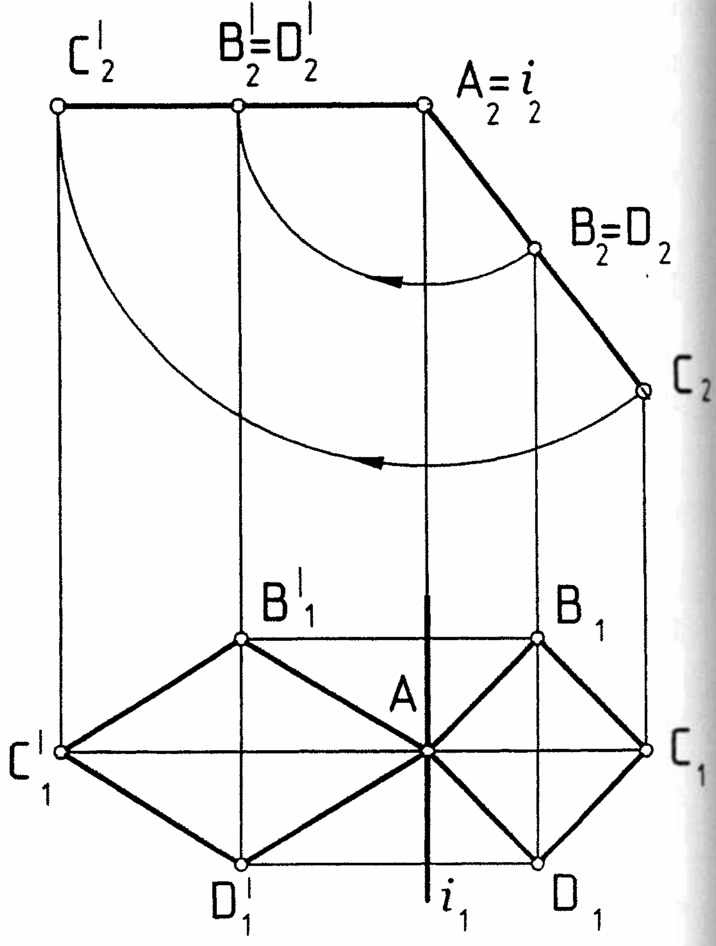


Fig.38

Questions for self-control

1. What are the subject and the aim of Descriptive Geometry and Drawing?
2. Give the definition of a complicated drawing.
3. Could you name conditions of communication between projections of the point on the complicated drawing?
4. What is a level line? Name level lines.
5. What is a projection line? Name projection lines.
6. What are competitive points?
7. What is a general position plane?
8. What is a projecting plane? Could you name projecting planes?
9. What is a level plane? Could you name level planes?
10. Name the condition of belonging of a point to a line.
11. Name the condition of belonging of a straight line to a plane.
12. What drawing is called reverse? What is a reverse drawing?
13. What are polyhedrons? What polyhedrons do you know? Name them.
14. How is the visibility of points and of lines lying on the face or on the edge of polyhedrons defined?
15. How is the rotation surface formed?
16. What rotation surfaces depending on the type of generating line (a straight line or a curve line) do you know?
17. How are the straight circular cylinder, a right circular cone and a sphere formed?
18. What is the difference in constructing a closed and opened (or circular) torus surface?
19. Name rotation surfaces of the second order.
20. What is the order of the torus surface?
21. What is the gist of the Method of Replacing Projection Planes? What problems may be solved by this method?
22. What does the Method of Revolution consist in? What problems may be solved by this method?
23. What new knowledge and skills should be acquired while studying the course of Descriptive Geometry and Drawing?
24. What teaching disciplines do you learn at the Graphical department?

CHAPTER II

RELATIVE POSITION OF GEOMETRICAL OBJECTS. DEFINITION OF THEIR COMMON ELEMENTS (POSITION PROBLEMS)

Problems in which relative position or common elements of geometrical objects are defined are called position problems. They also include the above mentioned problems connected with the point belonging to a line or with the line belonging to the surface.

§1. Relative Position of Lines

Two lines in space can be parallel, intersecting and skew.

1. Parallel Lines. The rule for making a complicated drawing of parallel lines is based on the fourth property of orthographic projecting, i.e. projections of parallel lines are parallel.

If lines in space are parallel, their projections are parallel. In Fig. 39 $a \parallel b \Rightarrow a_1 \parallel b_1 \wedge a_2 \parallel b_2$. For lines of general position the converse statement is true: $a_1 \parallel b_1 \wedge a_2 \parallel b_2 \Rightarrow a \parallel b$.

Thus, it is necessary to have any pair of projections of each line in order to tell from the drawing about the parallelism of two lines of general position. For example, horizontals h и h' (Fig. 39, *b*) are parallel as their projections h_1 и h_1' are parallel too; the profile line-segments $[AB]$ и $[CD]$ (Fig. 40, *a*) are parallel as $[A_3B_3] \parallel [C_3D_3]$; the line-segment $[EF]$ is non-parallel to $[KL]$ as the line-segment $[E_3F_3]$ is non-parallel to $[K_3L_3]$ (Fig. 40, *b*).

2. Intersecting Lines. The rule to construct a complicated drawing of intersecting lines is based on the sixth property of orthographic projecting, i.e. the intersection point of the lines is projected into the intersection point of their projections (Fig. 41):

$$c \cap d = K \Rightarrow c_1 \cap d_1 = K_1 \wedge c_2 \cap d_2 = K_2 ,$$

points K_1 and K_2 lying on the same communication line.

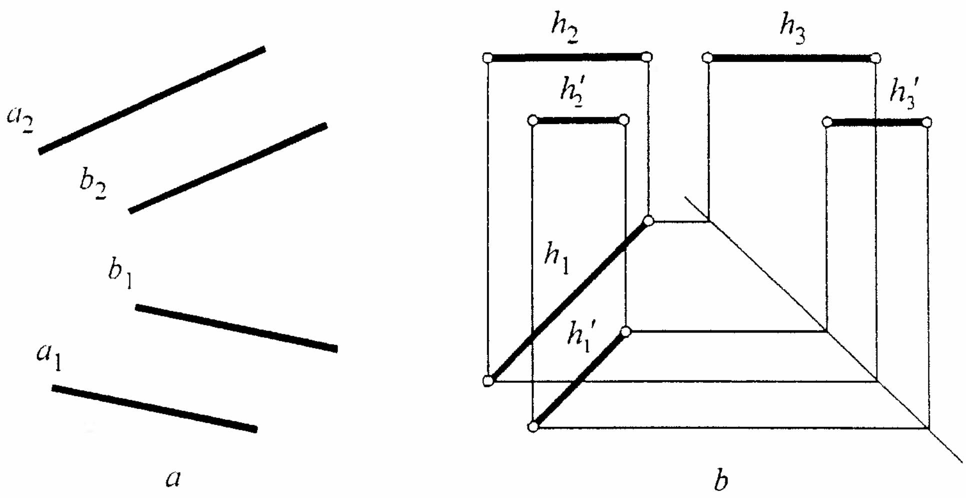


Fig. 39

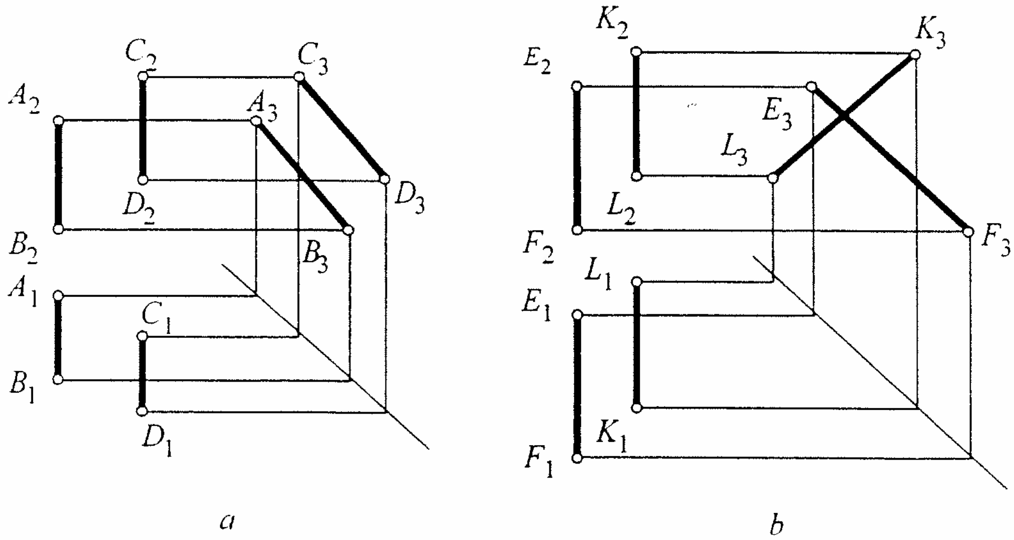


Fig. 40

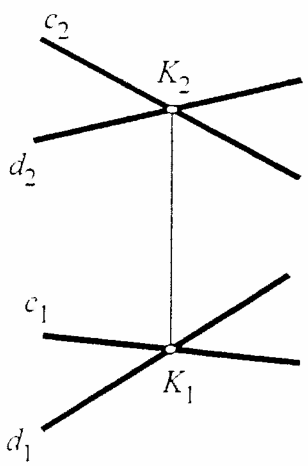


Fig. 41

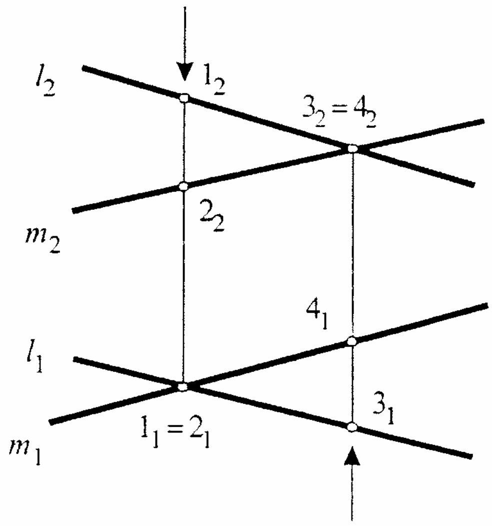


Fig. 42

3. S k e w L i n e s . Non-parallel and non-intersecting lines are called skew lines. One of possible variants of skew lines is shown in Fig, 42, where $l \div m$ as l is neither parallel nor intersecting m.

The intersection point of horizontal projections of skew lines is the horizontal projection of two horizontal competing points 1 and 2 lying on the lines l and m, respectively. The intersection point of frontal projections of skew lines is the frontal projection of two frontal competing points 3 and 4. The position of lines l and m relative to Π_1 is defined from horizontal competing points 1 and 2. The frontal projection 1_2 of the point 1 belonging to the line l is located higher than the frontal projection 2_2 of the point 2 belonging to the line m (the projection of the direction of viewing is indicated by the arrow). Hence, the point 1 is higher than the point 2 and the line l is located above the line m. The position of the lines l and m relative to the plane Π_2 is defined from frontal competing points 3 and 4.

The horizontal projection 3_1 of the point 3 belonging to the line l is located below the horizontal projection 4_1 of the point 4 belonging to the line m (the projection of the direction of viewing is indicated by the arrow). Hence, the point 3 is located in front of the point 4 and the line l is located in front of the line m.

§2. Definition of Common Elements of Geometrical Objects on the Basis of Condition of Belonging

P r o b l e m . Construct the intersection point of the line with the projecting plane.

Given the horizontal-projecting plane Γ and the straight line m of general position (Fig. 43). The point K of the intersection of the line m with the plane Γ belongs both to the line m and to the plane Γ. Hence, $K_1 \subset \Gamma_1 \wedge K_1 \subset m_1, \rightarrow K_1 = \Gamma_1 \cap m_1$; the point K_2 lies on the communication line on condition that $K_2 \subset m_2$.

P r o b l e m . Construct the intersection line of the general position plane with the projecting plane.

Given the plane Γ (ABC) of general position and the frontal-projecting plane Δ (Fig. 44). The required line k of the intersection with two planes Γ and Δ is a straight line, hence, it is defined by two points. To construct the line $k = \Gamma \cap \Delta$, it is required to find the points:

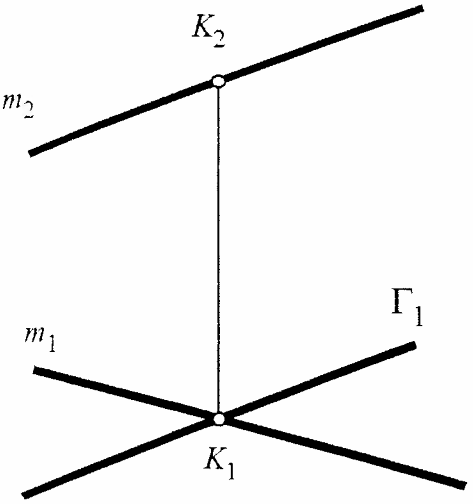


Fig. 43

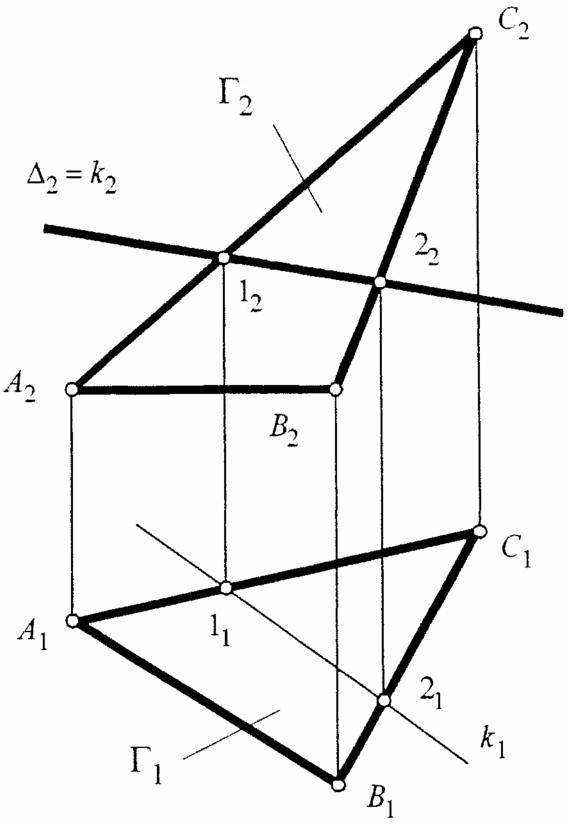


Fig. 44

$1 = AC \cap \Delta, \quad 2 = BC \cap \Delta.$

$1_2 = A_2C_2 \cap \Delta_2; \quad 1_1 \in A_1C_1; 2_2 = B_2C_2 \cap \Delta_2; \quad 2_1 \in B_1C_1.$

$k_1(1_1 – 2_1); \quad k_2(1_2 – 2_2) = \Delta_2.$

P r o b l e m . Construct the intersection line of two projecting planes.

Given two frontal-projecting planes: Δ and Γ (Fig. 45). The line of their intersection is the frontal-projecting line k.

$k_2 = \Delta_2 \cap \Gamma_2$, whereas k_1 coincides with the communication line.

P r o b l e m . Construct the intersection point of the projecting line with the general position plane.

Given the plane θ (KLM) of general position and the horizontal-projecting line g (Fig. 46). The point G of their intersection belongs to both the plane θ and the line g, hence it belongs to any line l drawn on the plane θ and intersecting the line g. In Fig. 46 $l \subset \theta \wedge l \cap g$. As $g \perp \Pi_1$, then $G_1 = g_1$, $G_2 = l_2 \cap g_2$.

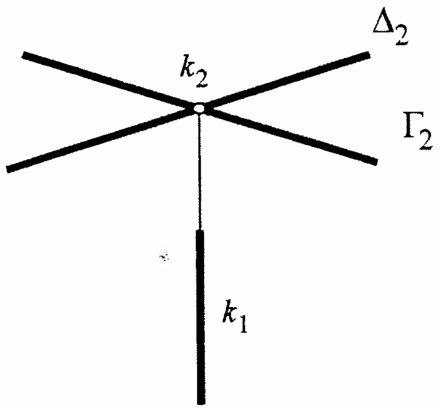


Fig. 45

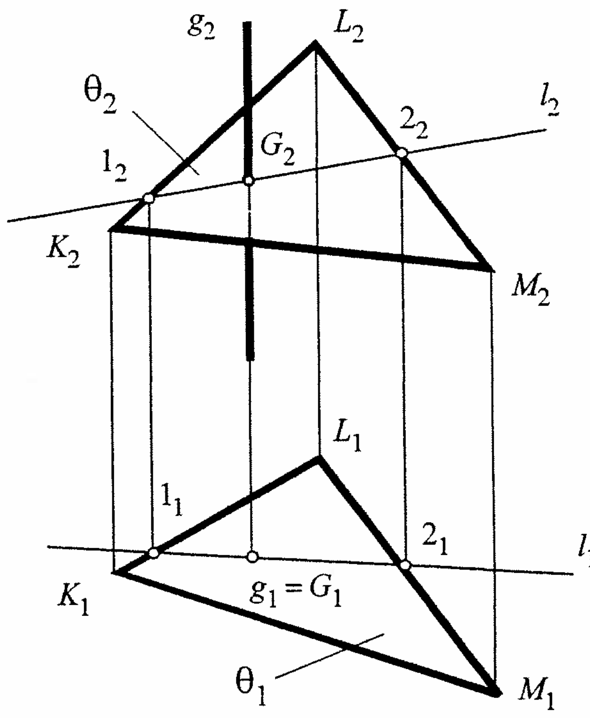


Fig. 46

§ 3. Intersection of Surfaces With a Projecting Plane and a Straight Line

The intersection line of a surface with a plane is a plane closed line. With a projecting cutting plane one projection of the intersection line coincides with the projection of the cutting plane, the other is constructed by points. Reference points are drawn first as well as points on the edges and then, if required, the intermediate points are drawn. The points obtained are joined on account of visibility and the character of the intersection line.

3.1. Intersection of a Polyhedron With a Projecting Plane

The intersection line of a polyhedron with a plane is a plane broken line, its vertices being points of edge intersection and the sides — intersection lines of polyhedron faces with the plane.

Fig. 47 shows the construction of the intersection line of a right hexahedral prism with the projecting plane Σ. Frontal projections $1_2, 2_2, 3_2$ ... of intersection points of prism edges with the plane are found at intersection points of frontal edge projections with the frontal projection of the plane. Horizontal projections $1_1, 2_1, 3_1$ ... of points coincide with horizontal projections of edges. Thus, the frontal projection of the required line is a line-segment $1_2 4_2$, the horizontal projection being the hexagon $1_1 2_1 3_1 4_1 5_1 6_1$. Profile projections of points are located along communication lines on profile projections of edges.

In Fig. 48 construction of intersection lines of the pyramid with the horizontal-projecting plane is shown.

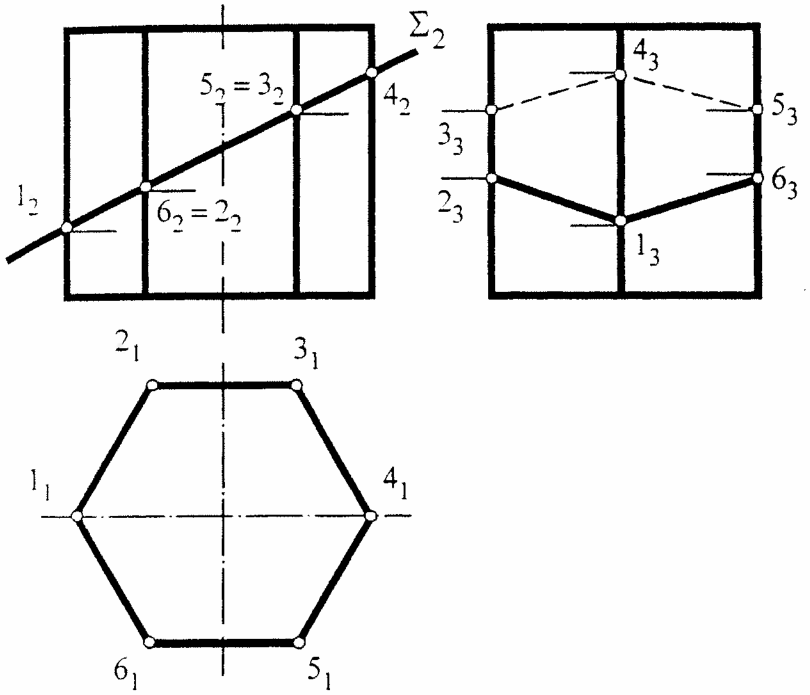


Fig. 47

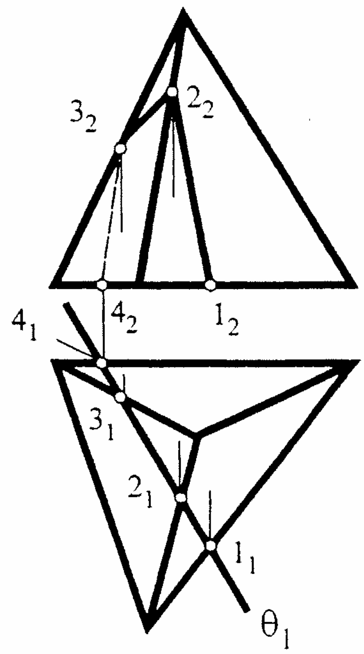


Fig. 48

3.2. Intersection of a Rotation Surface With a Projecting Plane

The intersection line of a curved surface with a plane is a plane-curved line.

To draw this curve it is required to define intersection points of several generatrices with the cutting plane. Points of control are the following: extreme points (the highest and the lowest relative to Π_1, the nearest and the farthest relative to Π_2), as well as the outline ones (their projections belong to outlines of horizontal and frontal projections of objects). In problems discussed they are also points of the visibility change.

Let's consider the intersection of the rotation cylinder with the plane.

If a cutting plane intersects all generatrices of the surface and is non-perpendicular to the rotation axis (Fig. 50, *a*), then the intersection line is an ellipse. In the drawing (Fig. 49) the plane Γ is not parallel to the axis *i* and intersects the cylinder across the ellipse. The ellipse is projected on the plane Π_2 as the segment $A_2B_2 = \Gamma_2$; whereas on the plane Π_1 — as a circle coinciding with the projection of the cylinder surface; and on the plane Π_3 — as an ellipse. Profile projections of points belonging to the ellipse are constructed by two given points (horizontal and frontal ones). Profile projections of the top and low points (*A* and *B*) are defined first followed by outline projections relatively to Π_3 (*C* and *D*) and then the intermediate ones, for instance, 1 and 2. Joining points defined by a smooth curve and taking into account the visibility? we then receive an ellipse which is the profile projection of a section object.

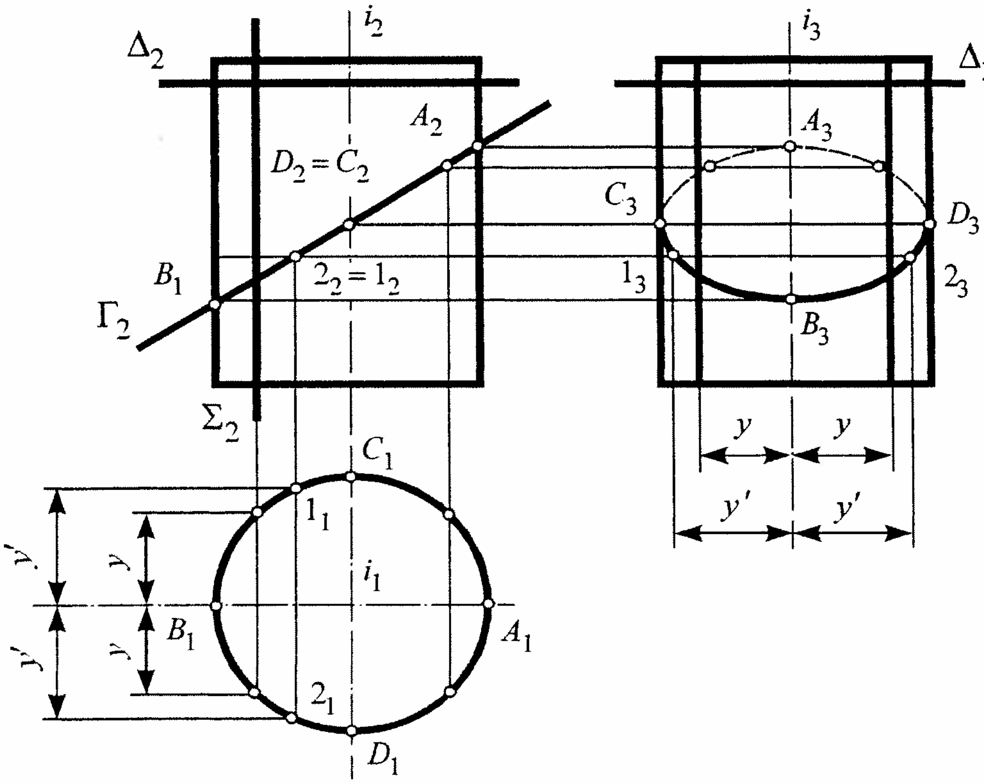
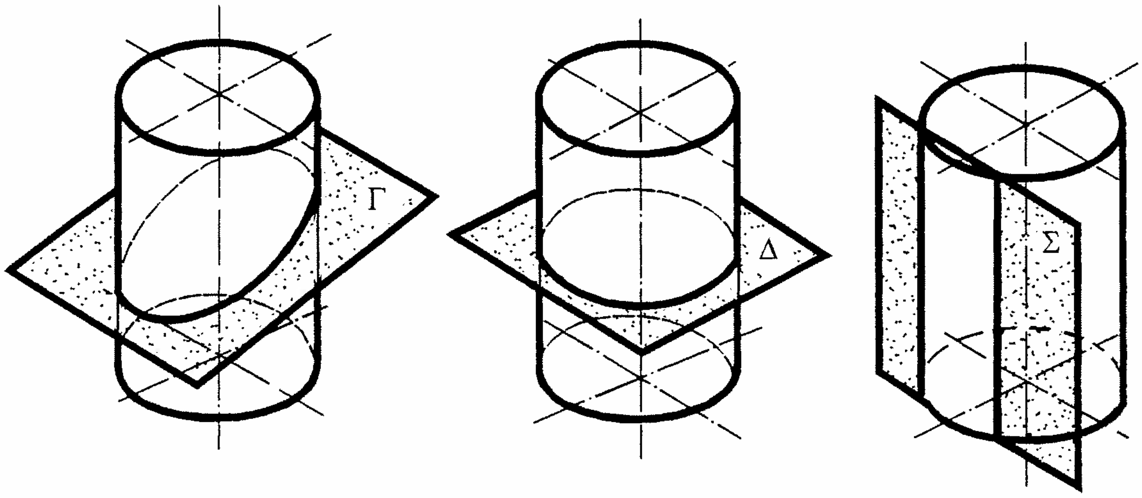


Fig. 49

If the cutting plane is perpendicular to the revolution axis (Fig. 50, *b*), then the intersection line is a circle.

In the drawing (Fig. 49) the plane Δ is perpendicular to the axis i and hence, it intersects the cylinder across the circle. On the plane Π_2 and Π_3 the circle is projected as line-segments coinciding with the corresponding projection of the cutting plane. On Π_1 the projection of the section circle coincides with the projection of the cylindrical surface.

If the cutting plane is parallel to generatrices of the surface (Fig. 50, c), then straight generatrices are obtained at the intersection. In the drawing (Fig. 49) the plane Σ is parallel to the axis i. Projections of generatrices on the plane Π_2 coincide with the plane projection, whereas projections on the plane Π_1 are projected as points belonging to the circle being the projection of the cylindrical surface. The third projection of generatrices is constructed by two projections (horizontal and frontal).



a *b* *c*

Fig. 50

In the intersection of the revolution cone with the plane all kinds of second-order curves can be obtained, such as an ellipse, a parabola and a hyperbola.

If the plane Σ (Fig. 51, *a*) intersects all generatrices of the cone, an ellipse is obtained in a section. In a particular case, when the plane is situated in Σ' perpendicular to the axis of the revolution cone, a circumference is obtained.

If the plane Σ is parallel to the generatrix l of the cone (Fig. 51, *b*), then the parabola is obtained in the section.

If the plane Σ is parallel to two generatrices l and l_1' of the cone (Fig. 51, *c*), then the hyperbola is obtained in the section. In a particular case, when the plane Σ transforming parallel to itself occupies the position Σ' (passes

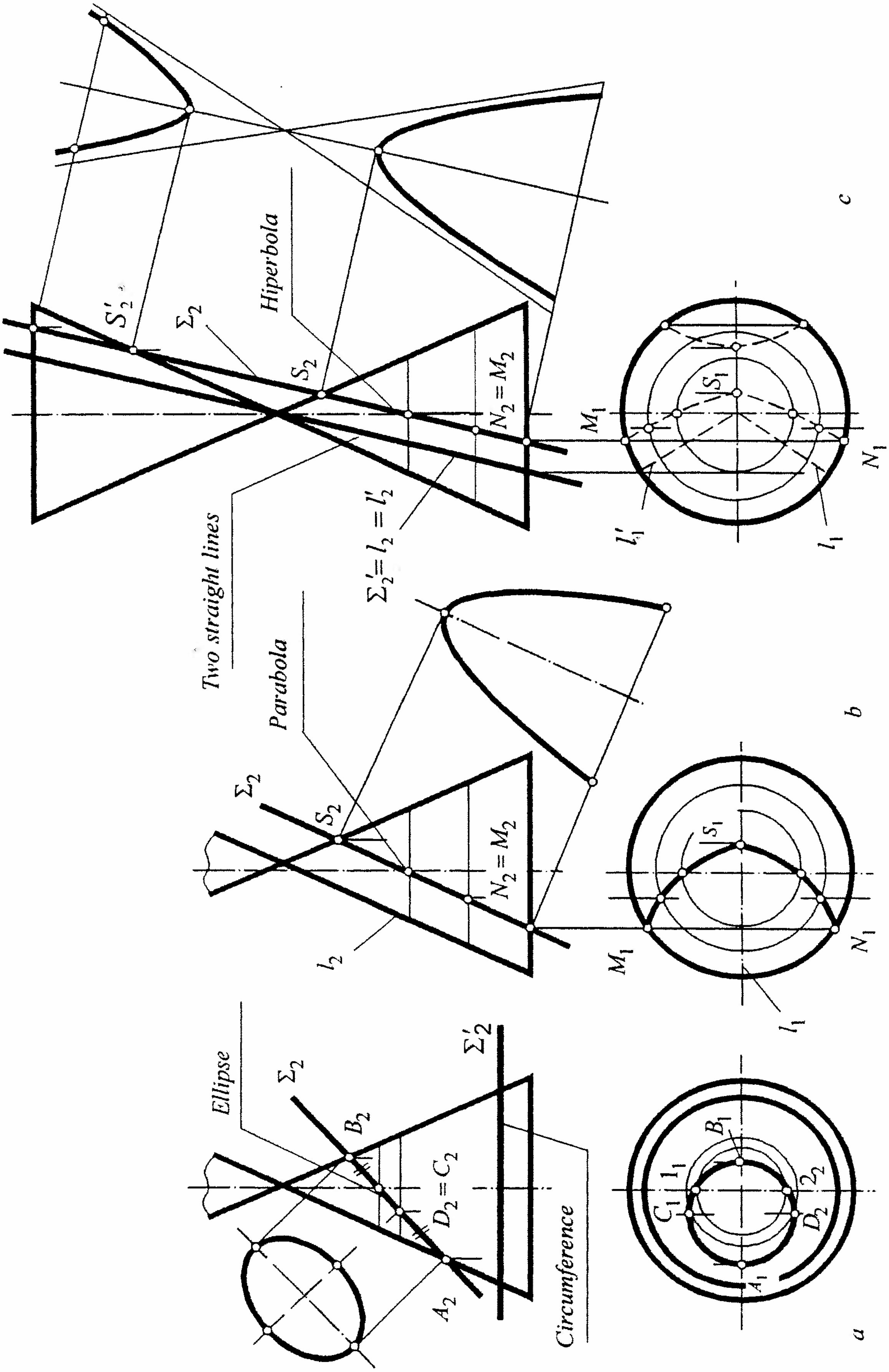

Ellipse
Circumference
Parabola
Two straight lines
Hiperbola
a
b
c

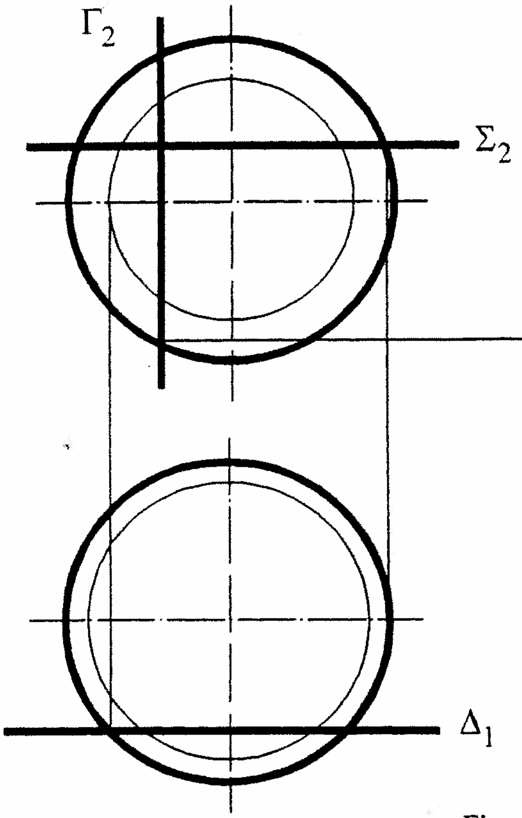
Fig. 51

through the vertex of the cone), the hyperbola degenerates into a pair of intercrossing lines (Fig. 51, *c*).

Frontal projections of the ellipse (Fig. 51, *a*), the parabola (Fig. 51, *b*) and the hyperbola (Fig. 51, *c*) on the cone surface coincide with frontal projections of cutting planes. Horizontal projections of points belonging to the ellipse, parabola and hyperbola are constructed with the aid of either generatrices or parallels.

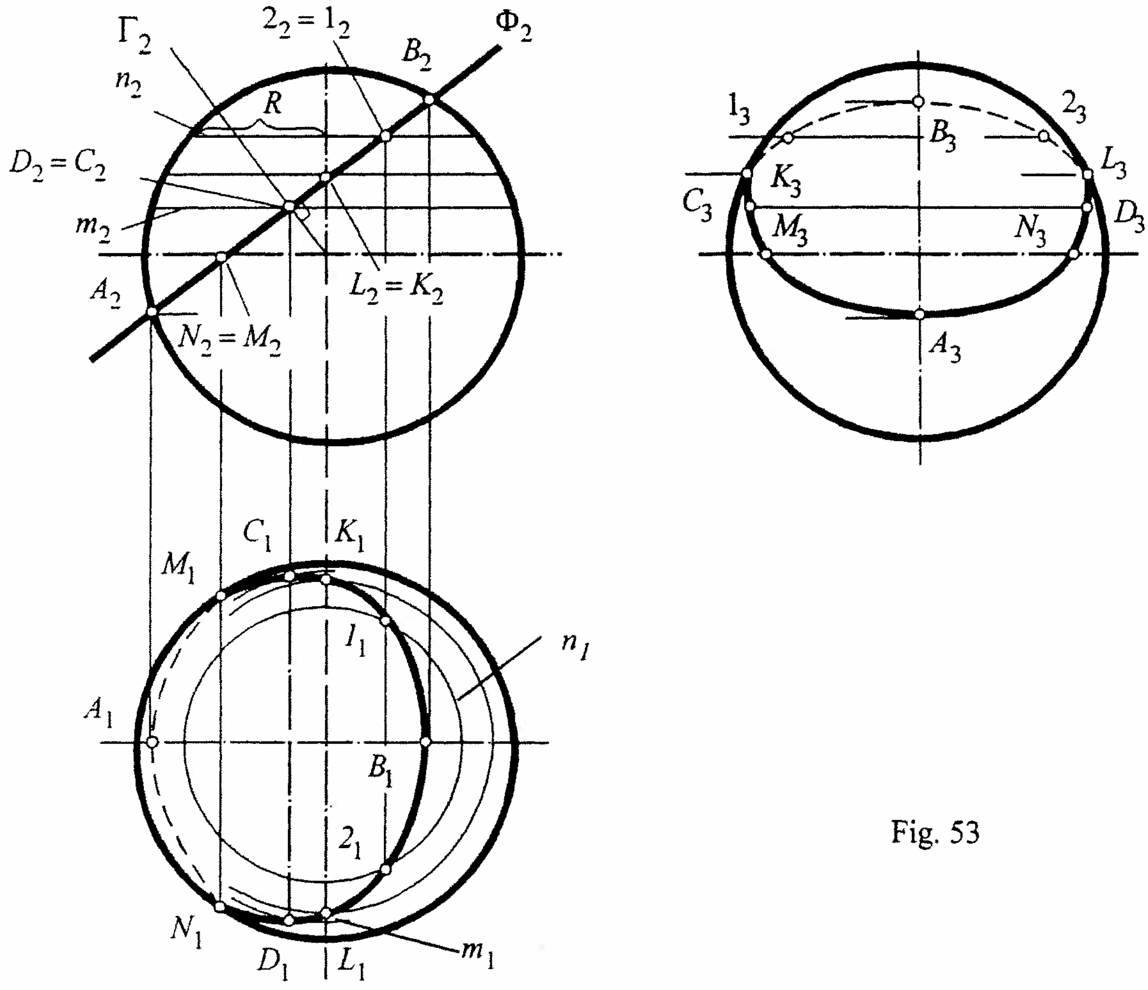
The plane intersects the sphere along the circle. Depending on the position of the cutting plane relative to the projection plane, the circle can be projected as a straight line, a circle (Fig. 52) or an ellipse (Fig. 53).

Fig. 53 shows the construction of intersection lines of the sphere with the frontal-projecting plane Φ. The circle of the section is projected on Π_2 as a line-segment $A_2B_2 = \Phi_2$, whereas on Π_1 — as an ellipse which is constructed by points. Points *A* and *B* are extreme relative to Π_1: *B* is the top point, *A* — the low one. Their frontal projections A_2 and B_2 coincide with intersection points of the frontal projection of the plane with the outline of the frontal projection of the sphere (the projection of the principal meridian). Their horizontal projections are defined by communication lines on the horizontal projection of the principal meridian. Frontal projections M_2 and N_2 of points *M* and *N* (points of the visibility change relative to Π_1) are markes at the intersection of Φ_2 with the frontal projection of the sphere equator. Their horizontal projections are defined by the communication line on the outline of the horizontal projection of the sphere (horizontal projection of the equator). Extreme points *C* and *D* relative to Π_2 (the nearest and the farthest) are defined with the aid of the general symmetry plane Γ, which is drawn through the centre of the sphere perpendicular to the plane Φ. Mark the frontal projections $C_2 = D_2$ of points *C* and *D* which coincide with intersection point Γ_2 and Φ_2. To define horizontal projections C_1 and D_1 of points *C* and *D* use the parallel *m* (m_1, m_2) passing through points *C* and *D*. Determine projections of intermediate points in the same way. For instance, projections of points 1 and 2 are constructed with the aid of the parallel *n* (n_1, n_2). On Π_2 this parallel is projected as a horizontal line intersecting the frontal projection of the intersection line at the point $1_2 = 2_2$. Construct the horizontal projection of the parallel — a circle with the radius *R*. In intersection points of this circle with vertical communication lines mark projections 1_1 and 2_1 of points 1 and 2.



Γ2
Σ2
Δ1

Fig. 52



Γ2
2₂ = 1₂
Φ2
B2
n2
R
D2 = C2
m2
A2
N2 = M2
L2 = K2
C1
K1
M1
1₁
n1
A1
B1
2₁
N1
D1
L1
m1
1₃
2₃
B3
L3
K3
C3
D3
M3
N3
A3

Fig. 53

3.3. Intersection of Surfaces With a Straight Line

Possible number of intersection points of the surface with a straight line corresponds to the surface order. For instance, the straight line can intersect the sphere, the cone or the cylinder, i.e, the second-order surfaces at two points. Intersection points of intersection of surfaces (irrespective of their type) with a straight line are constructed according to the following scheme (Fig. 54, *a*):

1. Pass an auxiliary cutting plane Δ through the line l;

2. Draw the line m of the intersection of this plane Δ with the given surface Φ;

3. Mark the required points A and B of the intersection of the given line l with the constructed intersection line m.

This scheme can be represented symbolically in the following way:

1. $\Delta \supset l$.

2. $m = \Delta \cap \Phi$.

3. $A = (l \cap m) \wedge B = (l \cap m)$.

As a rule, auxiliary cutting planes of a particular position give the simplest solution. While solving problems in intersecting a straight line with a curved surface, an auxiliary plane should be constructed in such a way that the lines the projections of which are straight lines or circles, would be obtained in a section. The visibility of projections of the straight line is found either by the visibility of the surface or by competing points.

P r o b l e m . Construct intersection points of the general position line with the pyramid (Fig. 54, *b*).

On the base of a general scheme a solution algorithm is developed. An algorithm is a number of unique, successive operations required to solve this problem. The scheme is transformed into an algorithm if the position of the auxiliary plane is shown accurately. Choose the horizontal-projecting plane Δ (as well as $\Delta \perp \Pi_2$) as an auxiliary plane.

The algorithm:

1) $\Delta \supset l$, $\Delta \perp \Pi_1$, i.e. draw the horizontal-projecting plane Δ through the line l ;

2) 1–2–3–4–1 $= \Phi \cap \Delta$, i.e. determine segments of the broken line of the intersection of the pyramid surface Φ with the plane Δ;

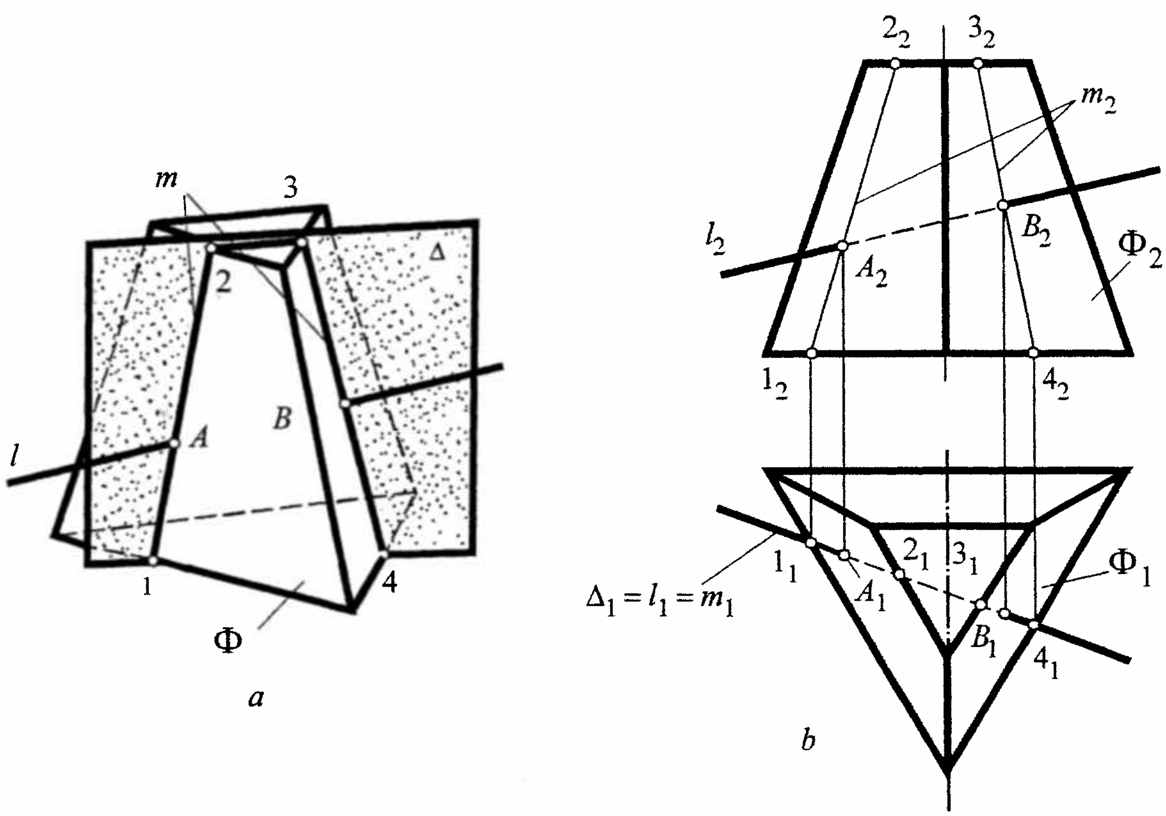


Fig. 54

3) $A = (1–2) \cap l$, $B = (3–4) \cap l$, i.e. mark the required points A and B of the intersection of segments (1–2) with l, (3–4) with l.

The graphic representation of the algorithm is given in Fig. 54, *b*.

P r o b l e m . Construct intersection points of the frontal and the sphere (Fig. 55).

In this problem it is required to use the frontal plane of the level $\Sigma \supset f$ as an auxiliary one, since the circle m of the section of the sphere θ is projected by the plane on Π_2 as a circle.

The algorithm:

1) $\Sigma \supset f$, $\Sigma \parallel \Pi_2$,

2) $m = \theta \cap \Sigma$,

3) $1 = m \cap f \wedge 2 = m \cap f$.

Making constructions on the drawing (Fig. 55), determine the visibility of intersection points and projections sections of the straight line. The point 1 lies on the frontal lower part of the sphere (visible on Π_2 and invisible on Π_1), therefore the frontal projection 1_2 of the point is visible, whereas the horizontal 1_1 is invisible. The point 2 lies on the frontal upper portion of the sphere (visible both on Π_2

and Π_1), therefore the frontal 2_2 and horizontal 2_1 projections of the point 2 are visible on the drawing.

P r o b l e m . Construct intersection points of projecting lines k and d with the cone surface (Fig, 56).

If the given straight line is a projecting one, then while solving the problem of its intersection with the surface auxiliary planes are not drawn. In this case projections of the required points are constructed on account of the principle of the point's belonging to the surface.

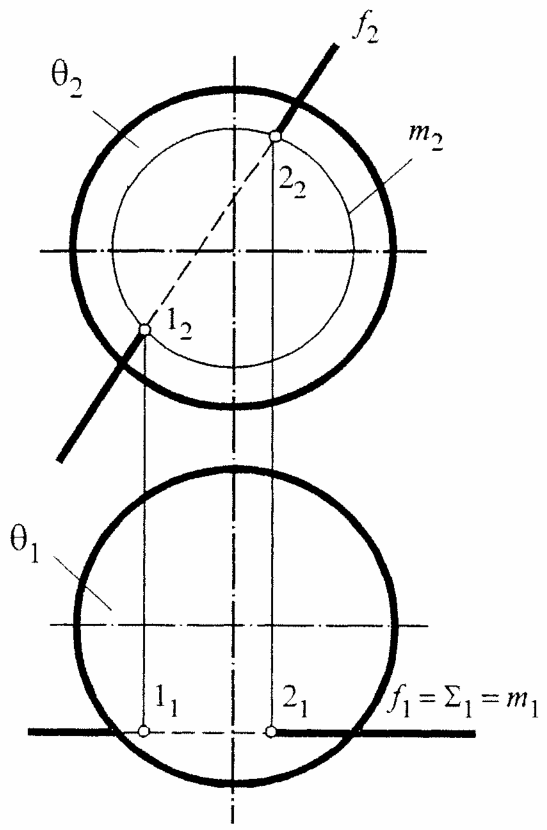


Fig. 55

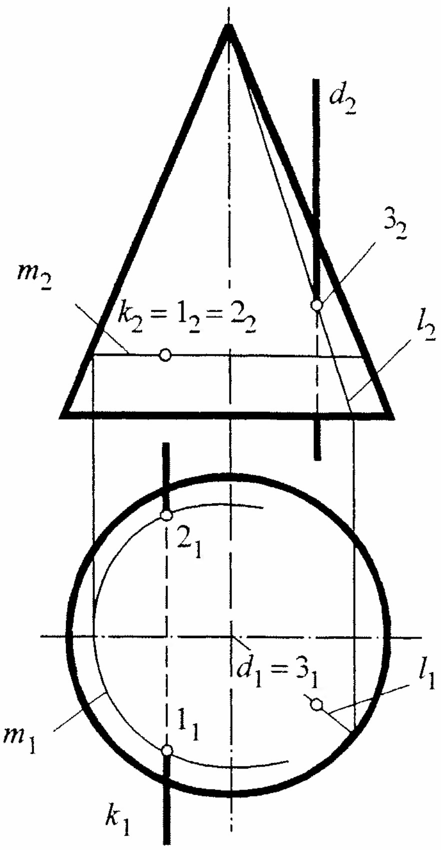


Fig. 56

In Fig. 56 the construction of projections of points 1 and 2 of the intersection of the line with the cone is carried out with the aid of the cone's parallel m to which all the required points belong. Projections of the point 3 of the intersection of the line d with the surface are constructed on account of the generating line l which belongs to the cone.

Fig. 57 shows the construction of intersection points of the straight line with projecting surfaces of the cylinder and the prism. The required intersection points are determined without additional constructions.

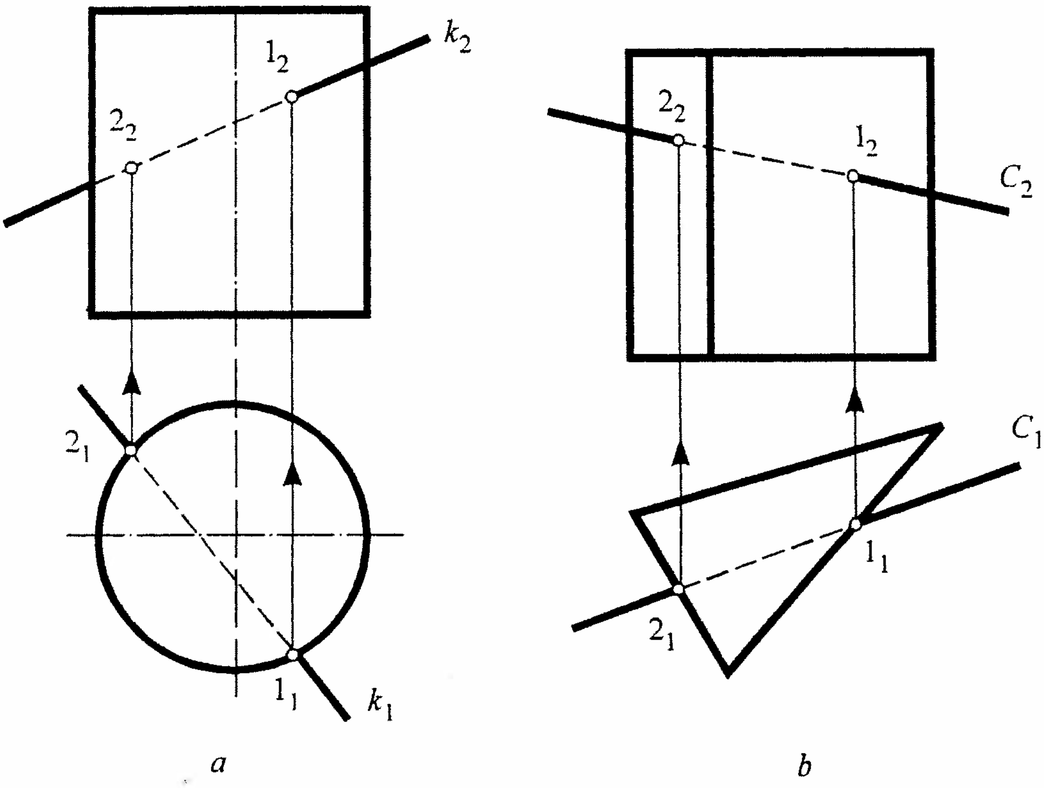


Fig. 57

§ 4. Construction of the Intersection Line of Surfaces

In engineering practice it is often required to make drawings of different parts the surfaces of which (polyhedron and curved) are interconnected.

In order to construct projections of lines of their intersection, it is necessary to be capable to analyze all possible cases of the intersection of surfaces as well as to know how to construct these lines.

The general line of two surfaces is called the line of their intersection (Fig. 58). Depending on the kind and the relative position of the surface this line can be a plane or a spatial broken line, a plane or a spatial curved line.

The intersection can be complete (penetration) and partial (cutting-in). In complete intersection (penetration) all generatrices lines or edges of one surface are intersected with the second surface. In this case the intersection line is disintegrated into two closed independent curved or broken lines (Fig. 61, 62, 63, 66, 67). In a partial intersection some generatrices (or edges) of one surface are intersected by some generatrices (or edges) of another surface. In this case the line of mutual intersection is a closed special curved or broken line (Fig. 60, 65, 68). The intersection line is constructed on the drawing by separate points, reference and intermediate ones belonging to the given surfaces simultaneously. To find points of the intersection line should be used either the principle of belonging or auxiliary surfaces: planes or spheres. In solving many problems both methods can be used at one and the same time.

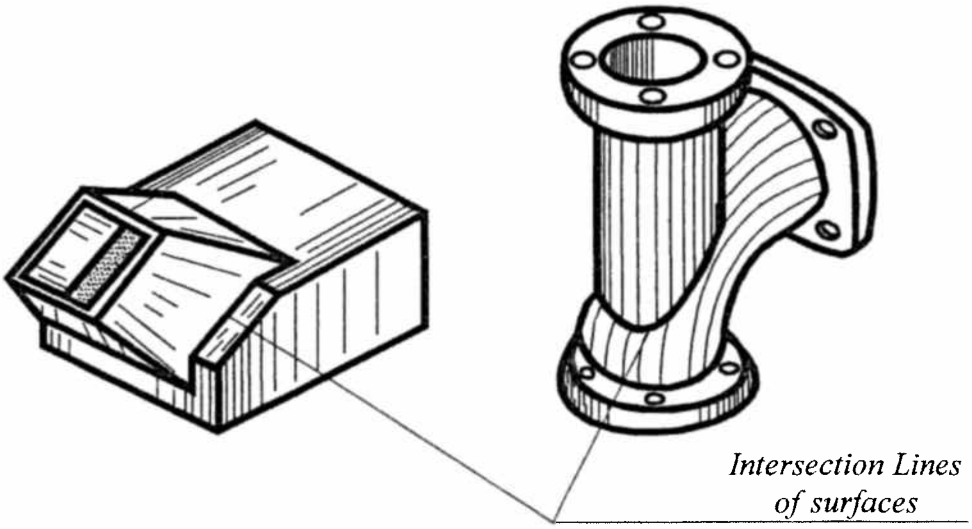


Fig. 58

4.1. Method of Auxiliary Planes

Method of auxiliary planes consists in the following (Fig. 59):

1. Construct the auxiliary plane Σ intersecting the given surfaces Φ and Ψ.

2. Determine lines *m* and *n* of the intersection of the auxiliary plane Σ with each given surface.

3. Mark points 1 and 2 of the intersection of the obtained lines *m* and *n* which are the required points as they belong simultaneously to the given surfaces.

The method can be represented symbolically in the following way:

1) $\Sigma \cap \Phi \wedge \Sigma \cap \Psi$,

2) $m = \Sigma \cap \Phi, \quad n = \Sigma \cap \Psi,$

3) $1 = m \cap n, \quad 2 = m \cap n.$

Multiple usage of the above method enables to determine the sufficient number of points (reference and intermediate) belonging to the intersection line of the given surfaces.

While solving problems in drawing intersection lines of surfaces auxiliary cutting planes should be chosen in such a way that they could intersect each given surface along the lines the projections of which are straight lines or circles.

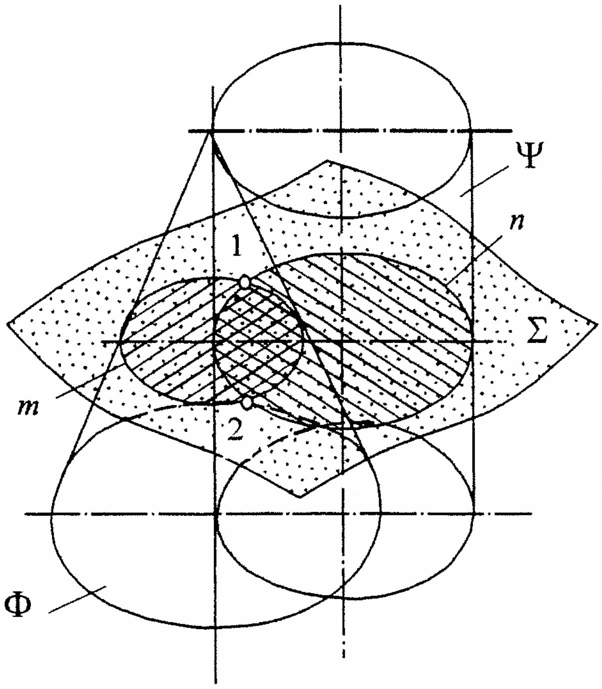


Fig. 59

If one of the given surfaces is a projecting one relative to any projection plane, then the respective projection of the intersection line coincides with the projection of this surface. In this case other projections of the intersection line can be determined on account of the principle of belonging.

Irrespective of the method of finding the required points of the intersection line (either by the condition of belonging or by auxiliary surfaces), the general procedure of solving problems on the surface intersection should be as follows:

1. Determine the type and the position of the given surfaces relative to each other and to the projection planes as well;

2. Determine the character of intersection lines (curved or broken lines, spatial or plane lines, etc.);

3. Determine reference points (points on edges, extremal and outline points);

4. Determine intermediate points;

5. Determine the visibility of projections of intersection lines and outline surfaces.

4.2. Construction of Intersection Lines of Polyhedral and Curved Surfaces

The intersection line of polyhedral and curved surfaces is a combination of several plane curves, each being the result of intersection of the curved surface with one of the polyhedral faces. These plane curves are intersected in pairs at intersection points of polyhedron edges with the curved surface. In a case of penetration this combination of plane curves is disintegrated into two or more parts. Separate line sections obtained at the intersection are curves: ellipse, hiperbola, parabola, circle, etc. Reference points are intersection points of polyhedron edges with the curved surface as well as extremal points and points of the visibility change.

P r o b l e m . Construct the intersection line of the straight triangular prism with the cylinder (Fig. 60).

The problem should be solved in the sequence recommended (see Section 4.1.).

1. The side faces of the prism are horizontal-projecting planes, the cylinder axis being perpendicular to the plane Π_3, i.e. the cylindrical surface is a profile-projecting one.

2. It is clearly seen from the horizontal projection that the cylinder surface is intersected with two side faces of the prism that are inclined to the axis, therefore the intersection lines are ellipse sections. The intersection of surfaces is partial (cutting-in).

The intersection line is projected on the horizontal plane in the form of two sections coinciding with the projection of the prism side faces. On the profile plane it is projected in the form of the circle coinciding with the projection of the cylinder side face. It is no need to construct horizontal and profile projections of the intersection line since they have already been constructed on the drawing. The frontal projection of the intersection line is, in fact, ellipse sections that are constructed along the points (reference and intermediate).

3. Determination of reference points. The frontal edge of the prism intersects the cylinder at points 1 and 2, their frontal projections being determined from

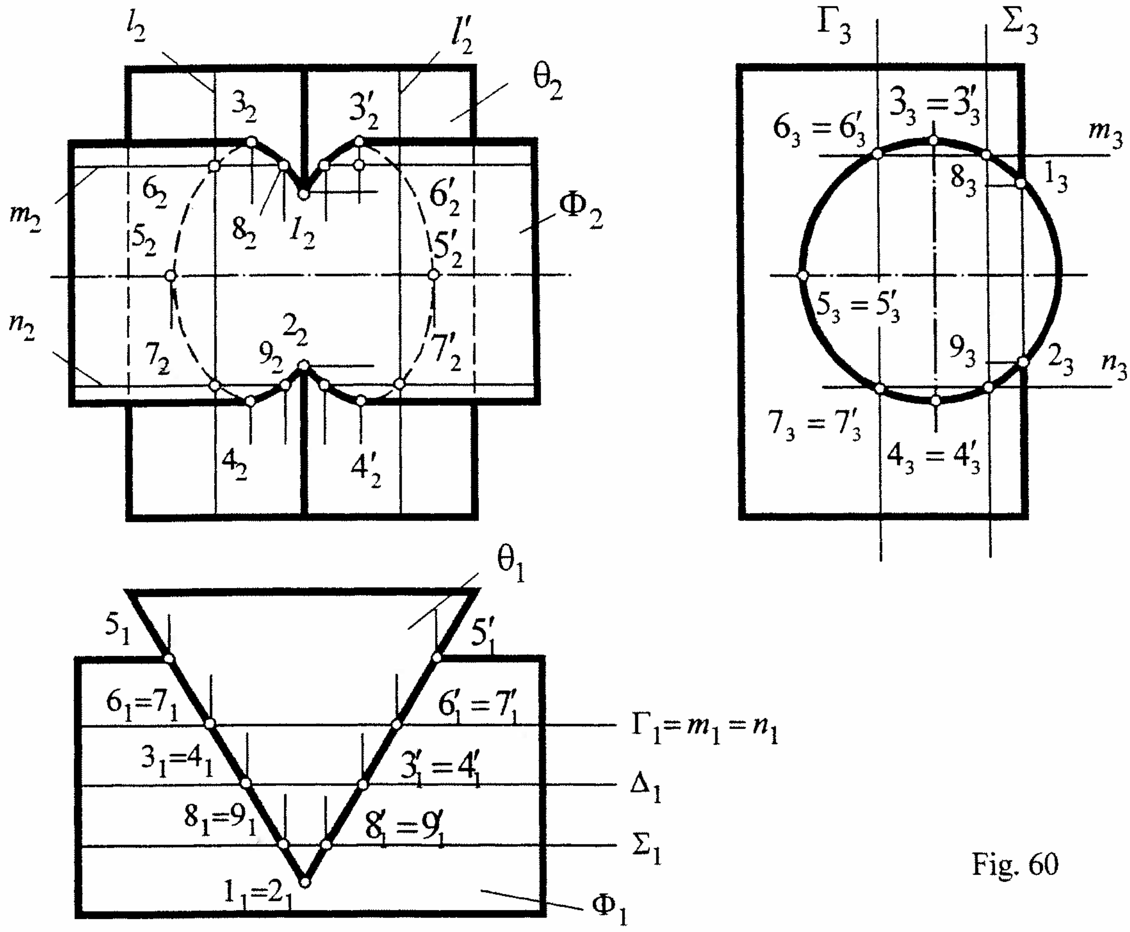


Fig. 60

the condition of belonging of these points to this edge. To construct the highest and the lowest points the auxiliary frontal plane of the level Δ is introduced which intersects the cylinder along the lines of the visible outline relative to Π_2, whereas it intersects the prism along the straight lines. At the intersection of the obtained lines points 3 and $3'$, 4 and $4'$ are marked which are at the same time points of the visibility change of the intersection line relative to the frontal plane of projections. Extremal points 5 and $5'$ relative to Π_2 are constructed from points belonging to the generatrix of the cylinder, which is the nearest to Π_2.

4. To make the most accurate construction of the intersection line on the drawing it is sufficient to find intermediate points. The algorithm for finding points 6 and 7, $6'$ and $7'$ is developed according to the general scheme (see section 4.1.):

1) $\Gamma \parallel \Pi_2$,

2) $\Gamma \cap \Phi = m \wedge n$; $\Gamma \cap \theta = l \wedge l'$;

3) $6 = m \cap l \wedge 7 = n \cap l$;

$6' = m \cap l' \wedge 7' = n \cap l'$

When the algorithm is developed, the construction is being made on the drawing (Fig. 60).

5. Frontal projections of reference and intermediate points obtained are connected by two smooth curves on account of visibility. Sections $3_2 - 1_2 - 3'_2$ and $4_2 - 2_2 - 4'_2$ are visible on Π_2 as they belong to both the frontal part of the cylinder surface and to the side prism faces visible on Π_2.

Fig. 61 and Fig. 62 show the cylinder and the cone with through prismatic holes; Fig. 63 shows the ball with a prismatic cut.

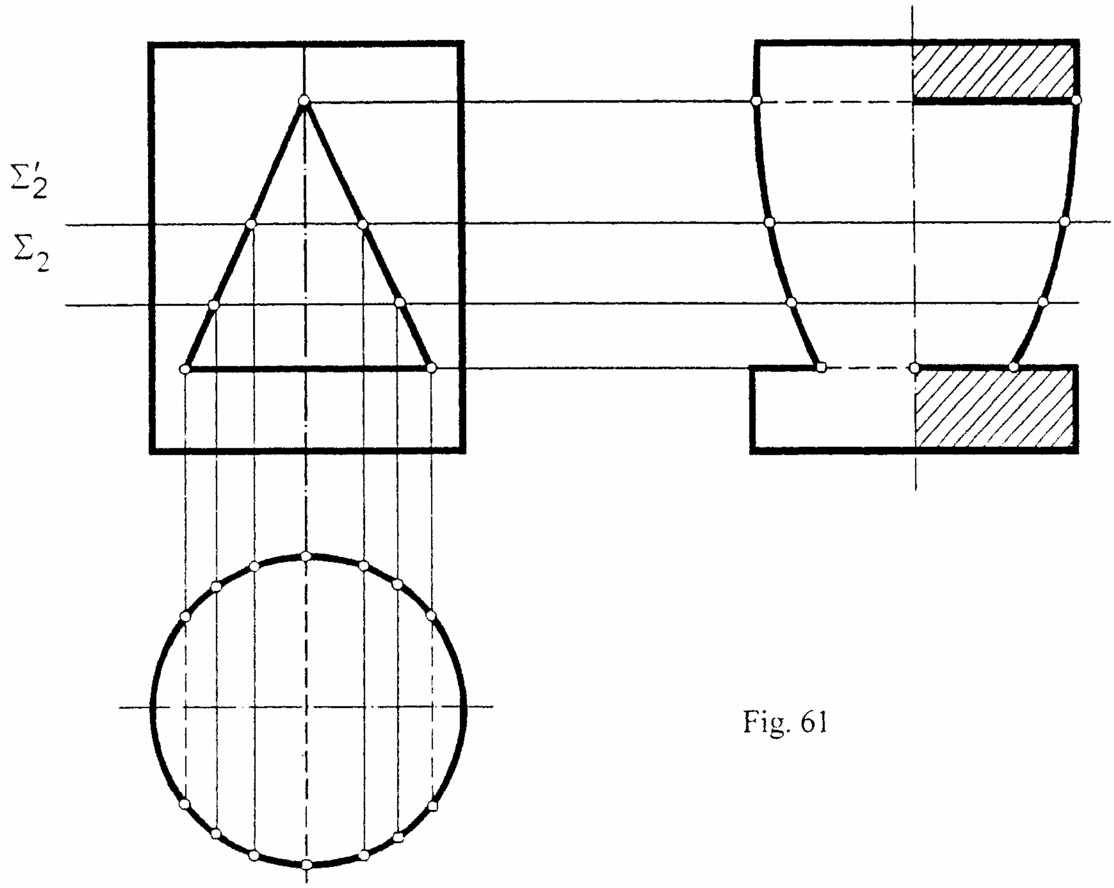


Fig. 61

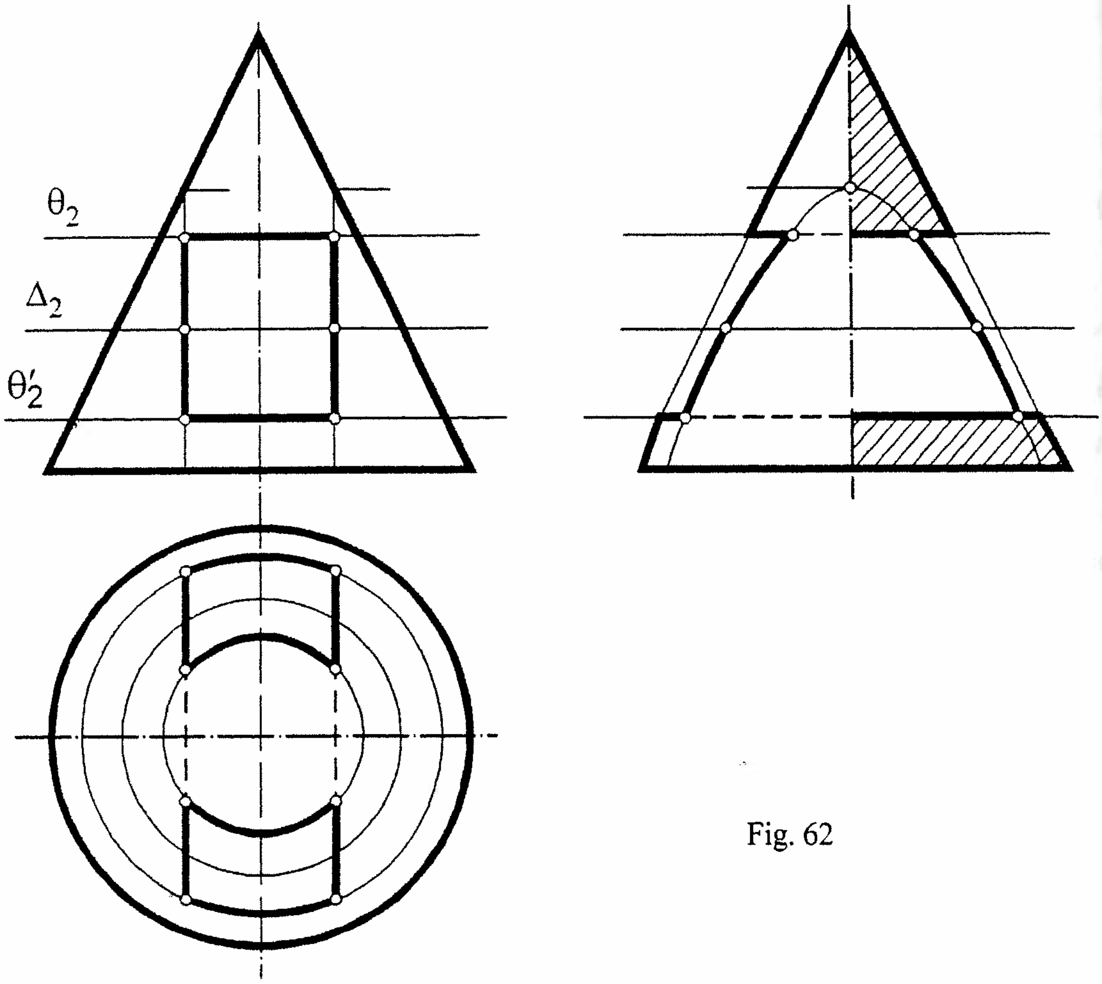


Fig. 62

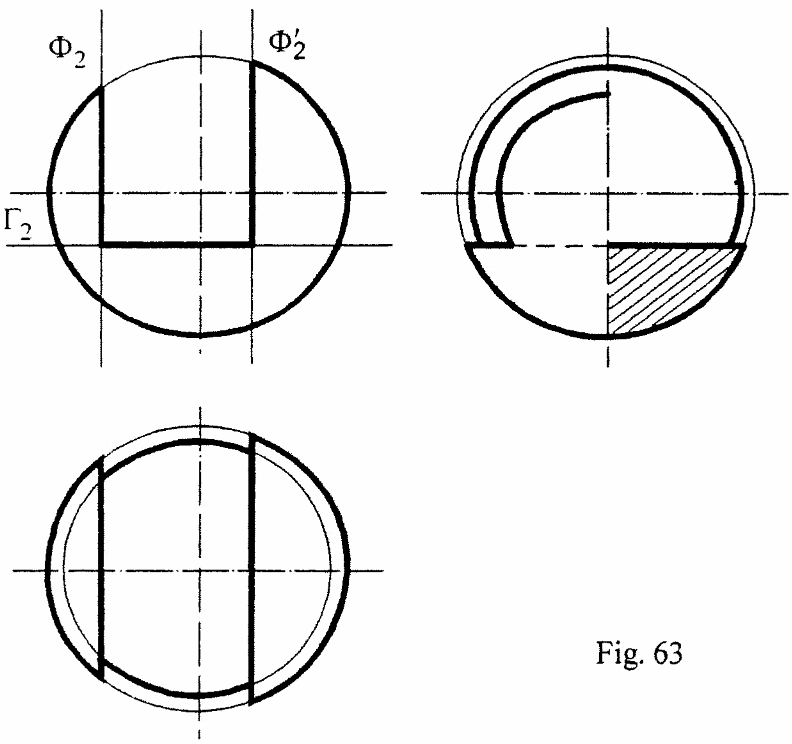


Fig. 63

4.3. Construction of Intersection Lines of Two Polyhedral Surfaces

Two polyhedral surfaces intersect along a closed spatial broken line (cutting-in) that can disintegrate into two closed broken lines (penetration). In all cases the vertices of the broken line are intersection points of the edges of the first polyhedron with the faces of the second one and of the edges of the second polyhedron with the faces of the first one, the sides being segments of a straight line along which the faces of both polyhedrons intersect (Fig. 64).

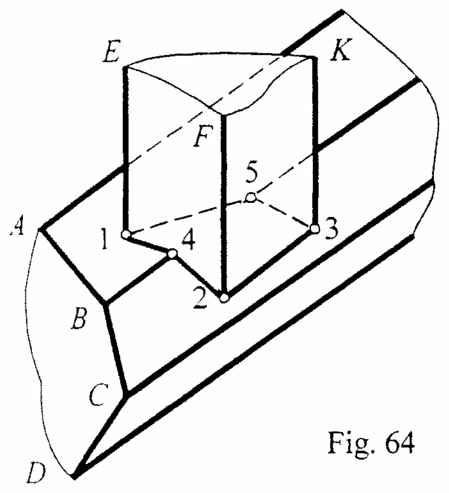


Fig. 64

Therefore the problem comes to the multiple construction of the intersection point of the straight line (edge) with the plane (face). When vertices (which are reference points) of the broken line are found, those pairs of vertices that belong to both one and the same face of the first polyhedron and to one and the same face of the second one are then joined by line segments on account of visibility.

P r o b l e m . Construct the intersection line of two right prisms (Fig. 65).

The intersection line of the prism is a spatial broken line, since the intersection is a cutting-in. The surface of the vertical prism is a horizontal-projecting one, whereas the surface of the horizontally located prism is a profile-projecting one. Therefore horizontal and profile projections of the intersection line coincide with corresponding projections of vertical and horizontal prisms in places where their projections are superposed.

Reference points are intersection points of vertical prism edges with horizontal prism faces (5, 6, 7, 8, 9, 10) and intersection points of horizontal prism edges with the faces of the vertical prism (1, 2, 3, 4). Mark their horizontal and profile projections and construct frontal projections on account of the condition of belonging of the points to the corresponding edges of both prisms.

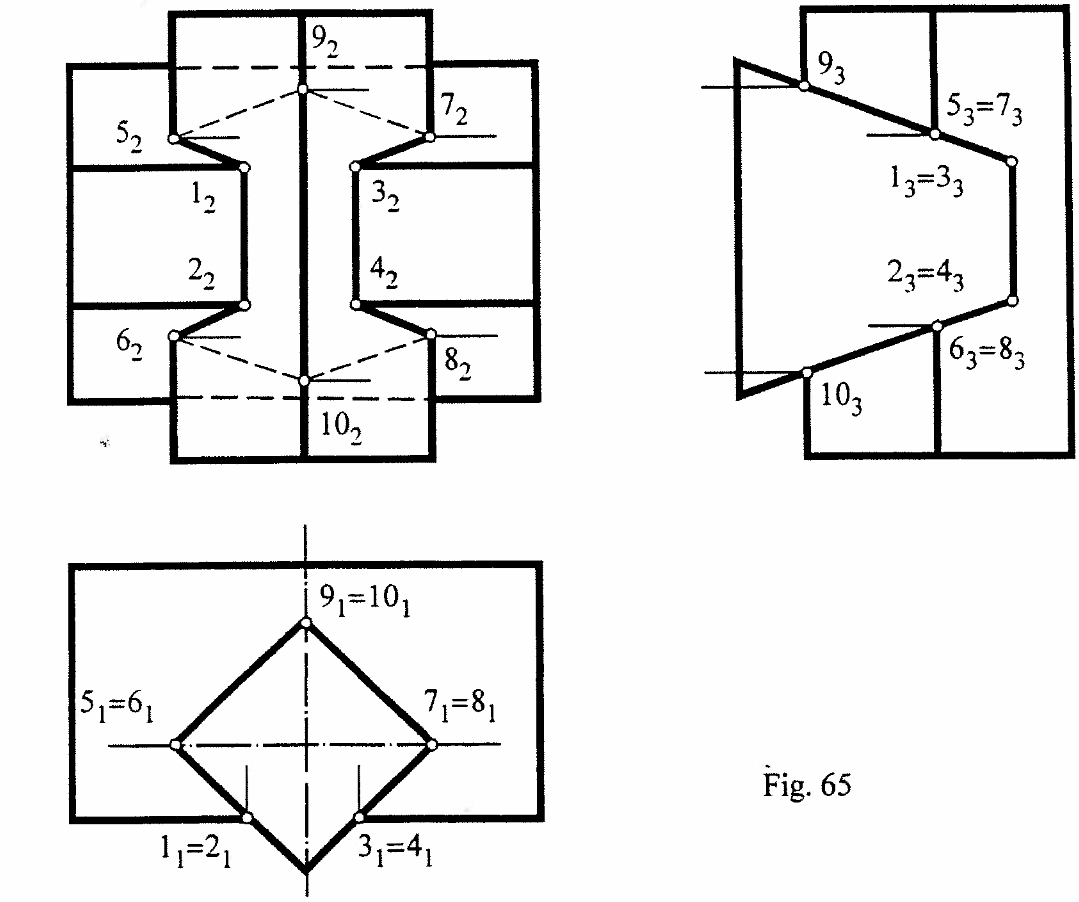

9_2
7_2
5_2
1_2
3_2
2_2
4_2
6_2
8_2
10_2
9_3
$5_3=7_3$
$1_3=3_3$
$2_3=4_3$
$6_3=8_3$
10_3
$9_1=10_1$
$5_1=6_1$
$7_1=8_1$
$1_1=2_1$
$3_1=4_1$

Fig. 65

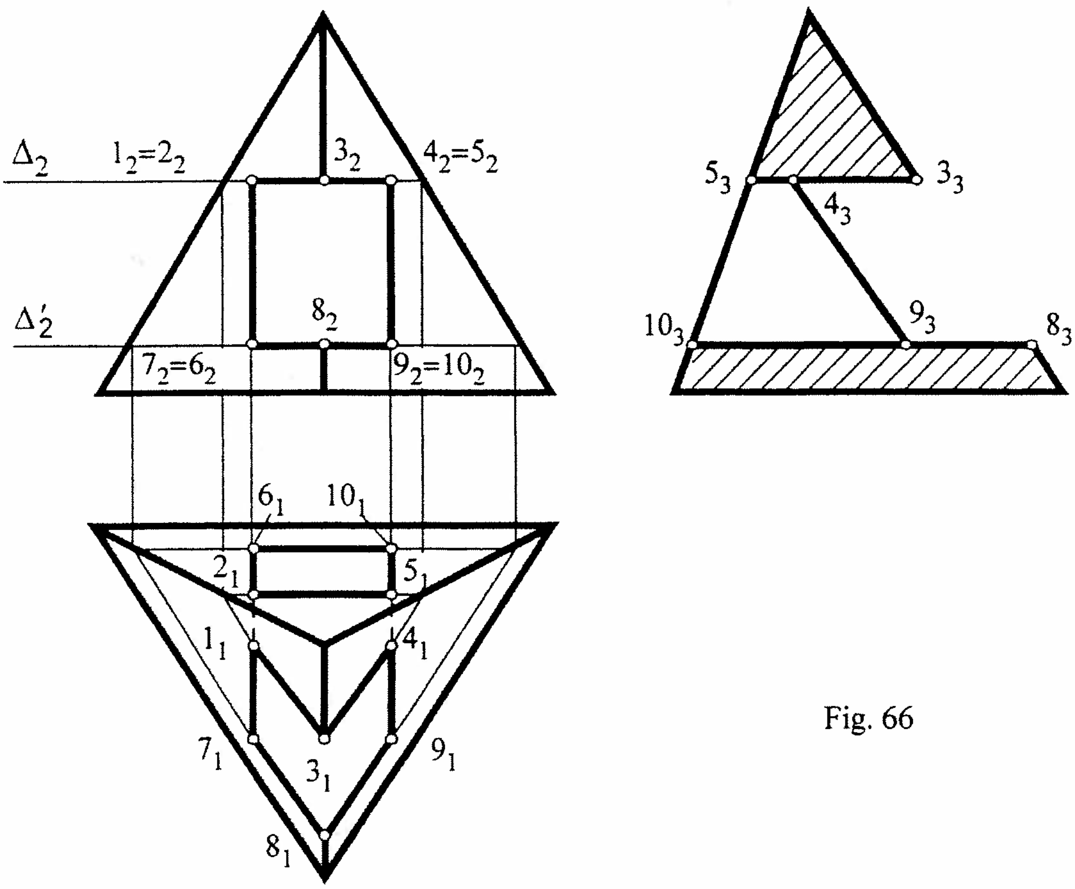

Δ_2
$1_2=2_2$
3_2
$4_2=5_2$
Δ'_2
8_2
$7_2=6_2$
$9_2=10_2$
6_1
10_1
2_1
5_1
1_1
4_1
7_1
3_1
9_1
8_1
5_3
3_3
4_3
10_3
9_3
8_3

Fig. 66

Using the above sequence, join points obtained by segments of straight lines. Elements 5–1–2–6 and 7–3–4–8 are visible relative to the frontal projection plane since they are resulted from the intersection of the given prism faces visible relative to Π_2.

Fig. 66 shows the image of a triangular pyramid with a prismatic through hole. The intersection is represented as penetration where intersection lines are disintegrated into a plane closed line (2–6–10–5–2) and a spatial one (1–7–8–9–4–3–1). While solving this problem the method of auxiliary cutting planes can be used.

4.4. Construction of Intersection Lines of Curved Surfaces

4.4.1. General Case

The intersection line of two curved surfaces in case of partial intersection (cutting–in) is represented as a spatial curve which in a complete intersection (penetration) can disintegrate into two or more segments. Reference and intermediate points of this line are determined by the method of auxiliary cutting planes, by the method of spheres or on the basis of the condition of belonging of the points to the surface. In a machine building drawing the most common case is the intersection of two cylindrical surfaces the axes of which are intersected at an angle of 90 degrees.

P r o b l e m. Construct the intersection line of two right circular cylinders the axes of which are perpendicular to the projection plane (Fig. 67).

On the drawing (Fig. 67) one can see penetration (two symmetrically located closed intersection lines). The surfaces of the given cylinders occupy the projecting position. In this case there is no need to construct two projections of the intersection line, since the profile projection coincides with the projection of the side surface of the small cylinder, whereas the horizontal projection — with the projection of the side surface of the large cylinder. Thus, it is sufficient to construct only the frontal projection of the intersection line.

Reference points — the uppest A, A', the lowest B, B', the boundary left C, C' and the boundary right D, D' — are determined on account of the condition of belonging of these points to generating lines of vertical or horizontal cylinders. The frontal projections of intermediate points 1_2, 2_2, 3_2, 4_2 and $1'_2$, $2'_2$, $3'_2$, $4'_2$ can be constructed on account of their belonging to their respective generatrices.

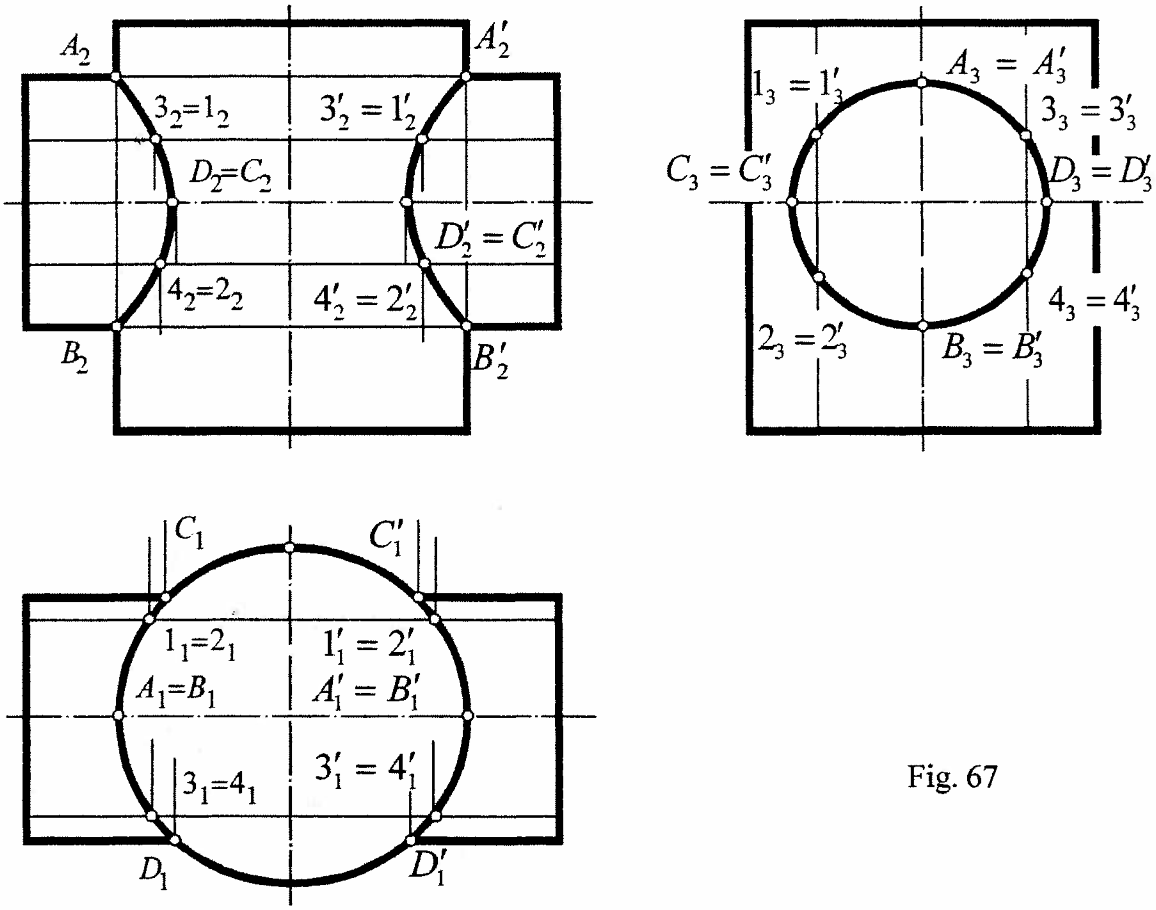


Fig. 67

Joining in sequence the frontal projections of the points obtained by a smooth curved line the frontal projection of the intersection line can be obtained. The projection of the visible line-segment located on the frontal portion of the cylinder coincides with the projection of the invisible line-segment.

P r o b l e m. Construct the intersection line of the cone with the cylinder (Fig. 68).

From the prescribed position of the surfaces on the complicated drawing cutting-in can be observed easily. Since the cylindrical surface is a frontal-projecting one, the frontal projection of the intersection line coincides with the frontal projection of the cylinder.

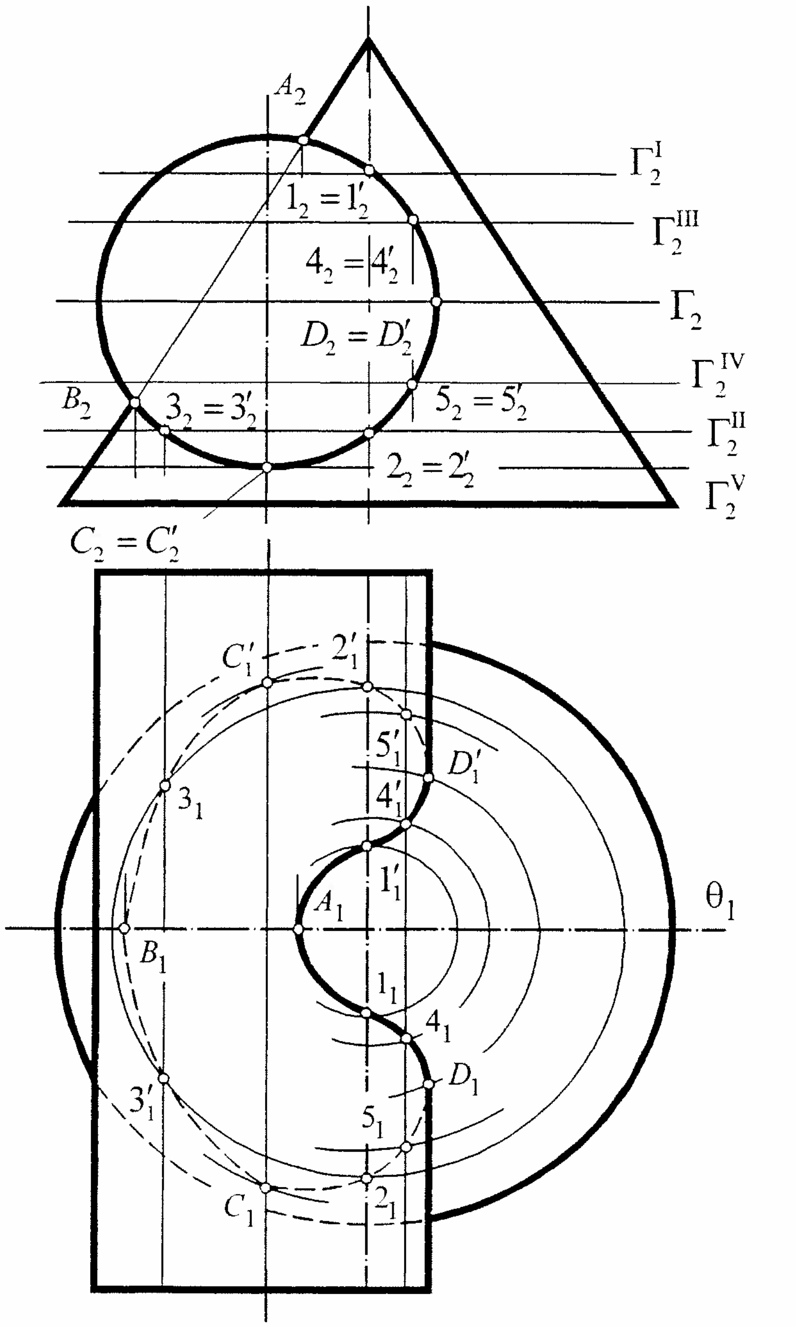


Fig. 68

To construct the horizontal projection of the intersection line it is sufficient to use auxiliary horizontal planes of the level Γ. These planes dissect the cone along the circle and the cylinder — along straight generating lines. Intersection points of these lines are projected into intersection points of projections of these lines.

While solving the problem first of all determine reference points: A and B — on the left-hand generating line of the cone (using the condition of belonging of the point to the surface), C and C' — extremal points (the lowest), D and D' — points on the right-hand generating line of the cylinder (points of the visibility change relative to the horizontal plane of projections). Intermediate points belonging to the intersection line are found by constructing auxiliary planes of the level symmetrically to the cylinder axis.

The points obtained are joined by a smooth curve on account of visibility.

4.4.2. Intersection of Coaxial Revolution Surfaces

Revolution surfaces having one common axis are called coaxial surfaces (Fig. 69). Meridians *m* and *n* of the coaxial revolution surfaces located on one axial plane Σ are intersected at points 1 and 2 (Fig 69, *a*). When meridians *m* and *n* revolve points 1 and 2, describe circles belonging to each of the revolution surfaces obtained. Thus, coaxial revolution surfaces are intersected along the circles. The number of the circles is equal to the number of the intersection points describing the surfaces of meridians located on one axial plane and on one side from the revolution axis (Fig. 69, *a*, *b*, *c*). For instance, axial revolution surfaces shown in Fig. 69, *a* are intersected along two circles since their meridians *m* and *n* have two

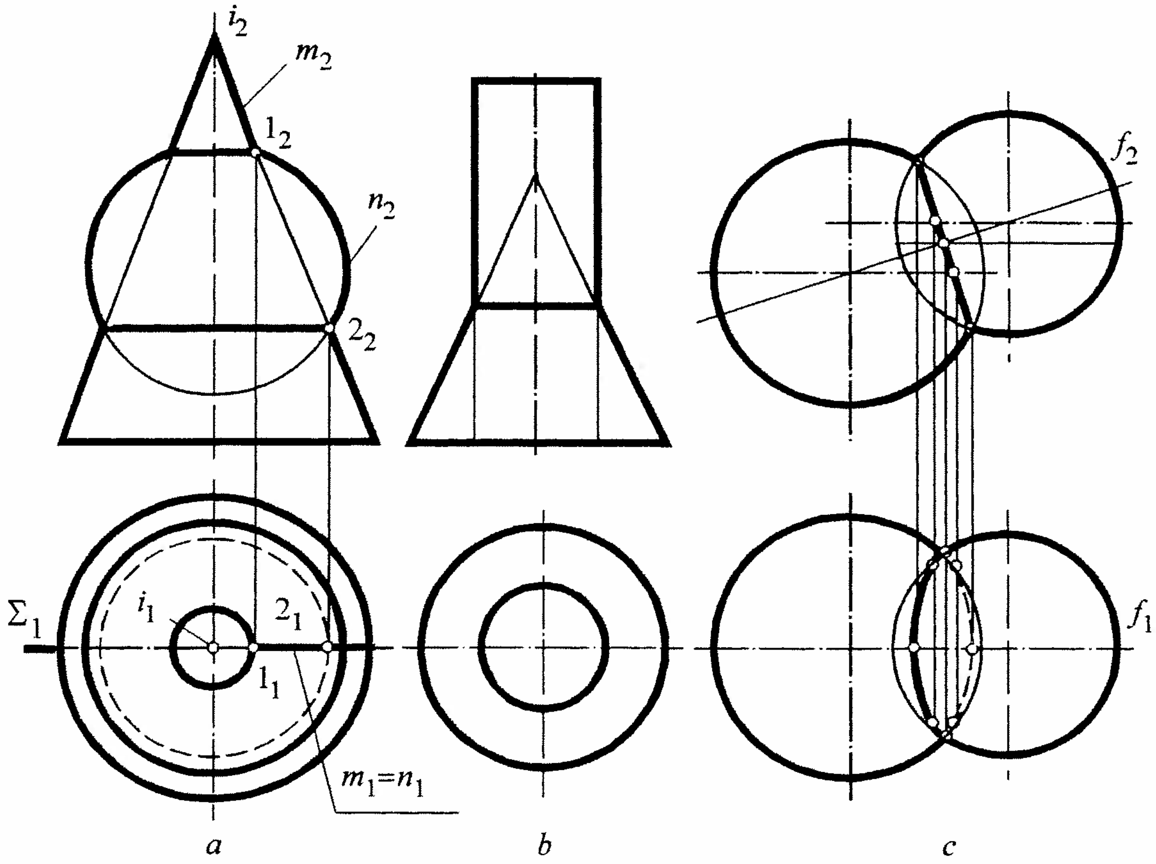


Fig. 69

common points: 1 and 2. Fig. 69, *c* shows mutual intersection of coaxial spheres, their common axis $f(f_1,f_2)$ being the frontal. In this case the intersection circle of the spheres belongs to the frontal-projecting plane and is projected on Π_1 as an ellipse, whereas on Π_2 it is a straight line.

Fig. 70 shows a blind-drilled hole with a conical facet. Surfaces forming the hole are coaxial revolution surfaces (truncated cone 1, cylinder 2, cone 3) which intersect along the circles. Projections of these circles on the frontal plane are straight line-segments.

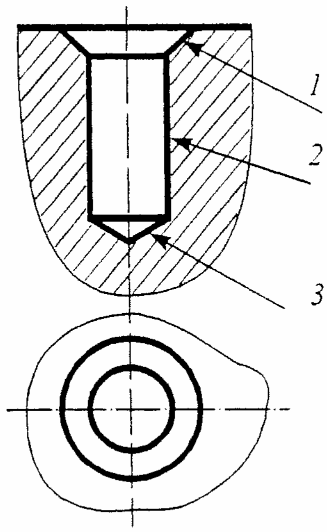


Fig. 70

4.4.3. Method of Auxiliary Spheres

The usage of auxiliary spheres in constructing the intersection line of curved surfaces is based on the property of coaxial revolution surfaces to intersect along the circles. The sphere with the centre in the point O of the intersection of axes of two revolution surfaces will be coaxial with any of these surfaces and will intersect them along circles l and m (Fig.71). Intersection points 1 and 2 of these circles are common to both surfaces, i.e. they belong to the line of their intersection. The construction of these points on the drawing (Fig. 71, *a*) is carried out easily since the plane of symmetry Σ of these surfaces is parallel to the frontal projection plane. Circles l and m are projected on Π_2 as line-segments l_2 and m_2. The point $1_2 = 2_2$ of their intersection is the frontal projection of points 1 and 2.

This method of constructing intersection surface lines is called the method of concentric spheres. To use this method some conditions should be observed, such as:

1) intersection of revolution surfaces;

2) axes of surfaces — intersecting lines are parallel to one of the projection planes, i.e. there is a common symmetry plane;

3) one cannot use the method of auxiliary cutting planes since they don't provide graphically simple lines on surfaces.

In order to find a series of points of the intersection line it is necessary to use spheres with various radiuses the centre of which is at the point of intersection of the axes of given surfaces. The minimum radius $|R_{min}|$ is equal to the radius of the largest sphere inscribed in these surfaces, while the maximum one $|R_{max}|$ is equal to the segment length denoting the distance between the projection of the sphere centre and the farthest intersection point of outline generatrices (Fig. 71, *a*).

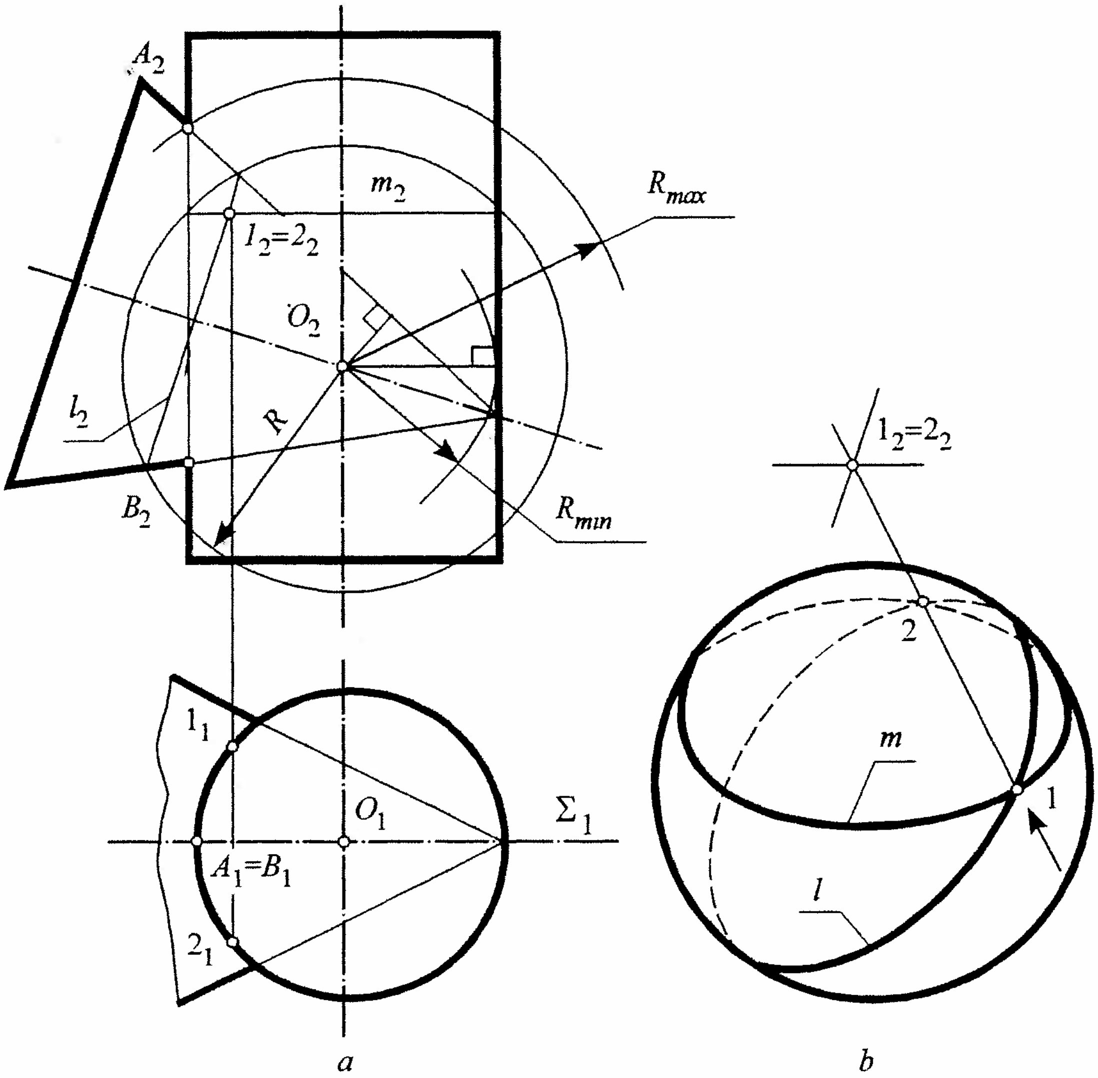


Fig. 71

The construction of the intersection line of the surfaces of the tore and the revolution cone by the method of concentric spheres is shown in Fig. 72.

Outline points *A* and *B* relative to Π_2 (the lowest) are found by means of the common plane of symmetry $\Delta \parallel \Pi_2$. The usage of auxiliary planes for the construction of other points doesn't provide a simple solution graphically. Since the axes of the given revolution surfaces intersect and are parallel to Π_2 (belong to the common plane of symmetry Δ), spheres with a common centre at the intersection point of the axes of the given surfaces can be chosen as auxiliary surfaces. The uppest points C and D (the nearest and the farthest relative to Π_2) are found by means of the sphere with the minimum radius inscribed in the tore. Intermediate points 1 and 2 are found by means of the sphere with the radius R, greater than $|R_{max}|$ and less than $|R_{min}|$.

When axes of intersecting surfaces skew but do not intersect the method of concentric spheres can't be used.

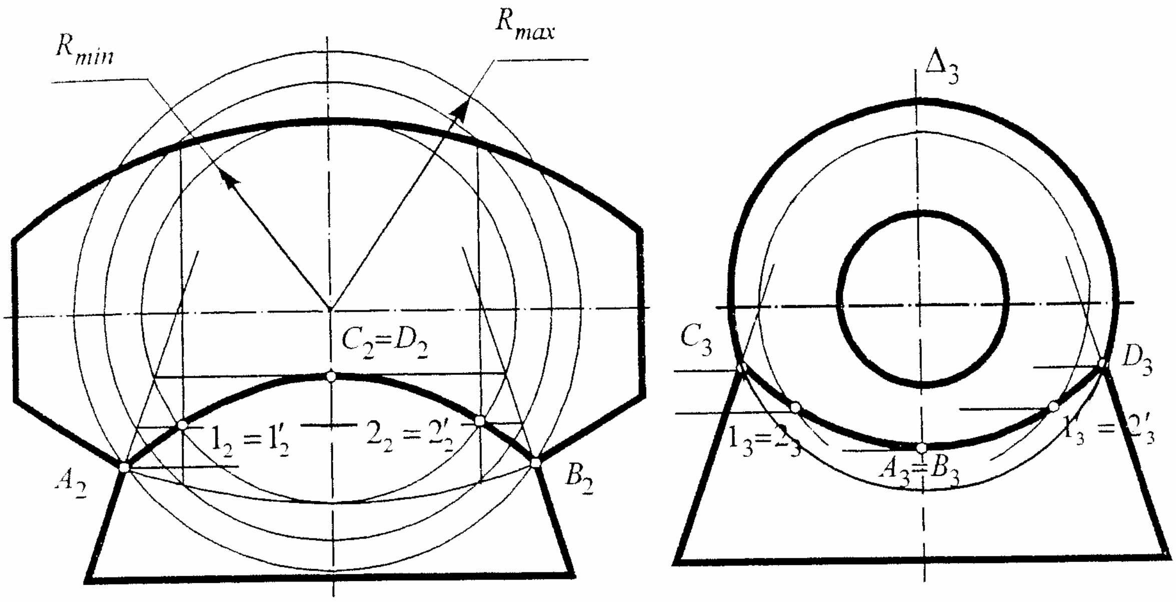


Fig. 72

But the method of eccentric sphere can be used when:

1) each of the surfaces has circular sections;

2) there is a common plane of symmetry parallel to one of the projections planes.

Fig. 73, *a*, *b* demonstrates surfaces of the tore and the cone having the family of circular sections. They can be intersected by the sphere along each of the sections. The tore is intersected by planes Γ, Γ′, ... passing through the revolution axis *i* along the circle *l*, *l′*, ... The geometrical place of the sphere centres providing circular tore sections is a perpendicular drawn from the centre of this circle to its plane (Fig. 73, *a*). The revolution cone is intersected by planes Λ, Λ′, ... parallel to the base along the circle *m*, *m′*, ... The geometrical place of the sphere centres providing these circular sections is the revolution axis the cone (Fig. 73, *b*).

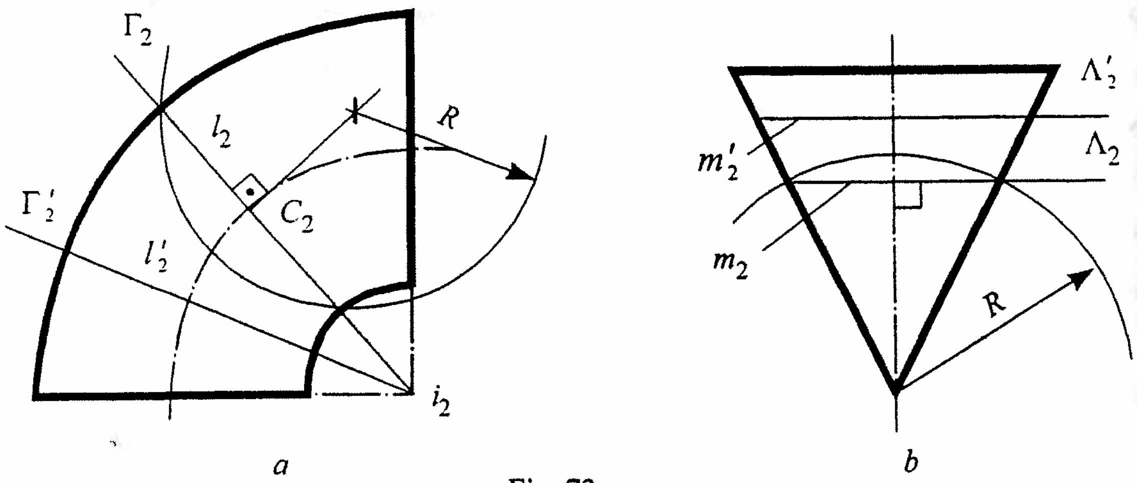


Fig. 73

To construct the intersection line of the tore and the cone surfaces (Fig. 74) eccentric spheres can be used. The centre of each sphere intersecting both the tore and the cone along the circles is at the point O of the intersection of the perpendicular drawn from the circle centre C to the plane of the circular section l and the cone axis.

Construct the sphere with the centre at the point O of the radius R, so that it would intersect the tore along the constructed circle l. This sphere will intersect the cone along the circle m. Intersection points 1 and 2 of these circles belong to both surfaces. Other centers O', O'', ... can be found similarly, as well as a sufficient number of points belonging to the intersection line can be

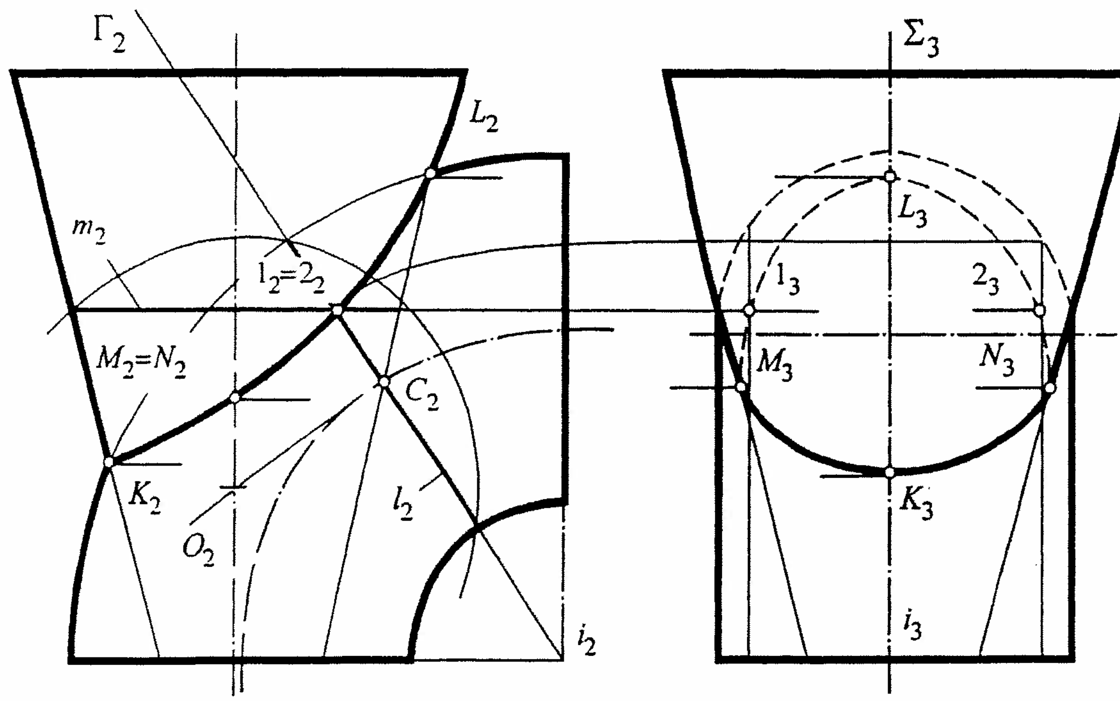


Fig. 74

constructed. The construction of extremely points K and L of the intersection line which are outline points M and N relative to Π_2 can be seen from the drawing.

4.5. Special Cases of Intersection of Second-Order Surfaces

The intersection line of two second-order surfaces in a general case is an algebraic curve of the fourth-order. In particular cases it can decompose into lines of lower orders, the sum of the orders being equal to four. Cases of its decomposition into the pair of curves of the second order are of a special interest. There are several theorems determining conditions of such decomposition. Let's analyze the Monzh theorem.

If two surfaces of the second order are described about the third one or are inscribed into it, their intersection line is disintegrated into two curves of the second order, their planes pass through the line connecting the intersection points of contact lines.

In Fig 75 the cylinder and the cone are described about one and the same spherical surface. According to the Monzh theorem their intersection line is decomposed into two plane curves — two ellipses the planes of which pass through the line KL joining intersection points K and L of the contact circles 1–2 and 3–4, as well as through points A, B, C and D, intersection points of outline generatrices.

Ellipses are projected on the plane Π_2 in the form of straight lines as the line $KL \perp \Pi_2$, while on the plane Π_1 it is projected in the form of ellipses the points of which are constructed on the basis of the condition of their belonging to the cone surface.

Fig. 76 demonstrates the intersection of cylinders having equal diameters (see the Monzh theorem).

Problem. Construct projections of intersection lines of surfaces forming the given object. Make sectional views.

The given object (Fig. 77) is composed of the cylinder with the vertical axis and two cones with horizontal axes. Inside there is a vertical cylindrical hole and a horizontal semi-cylinder with similar diameters, as well as two through horizontal cylindrical holes inside adjoining cones.

The problem is composed of four simple problems on constructing surfaces intersection lines .

Let's construct intersection lines of:

1) the vertical external cylinder with the adjoining cone having a horizontal axis (lines 1–5–3–6–2–6′–4–5′–1);

2) the horizontal semi-cylinder with an external cylindrical surface (line 7–9–8);

3) the horizontal semi-cylinder with internal vertical cylindrical holes having equal diameters (line 12–13);

4) the horizontal cylindrical holes in the cones with internal vertical cylindrical holes (lines 10–11– 10′–11′).

Each problem is solved on account of the theory of construction of intersection lines of surfaces in sequences recommended (see section 4.1.).

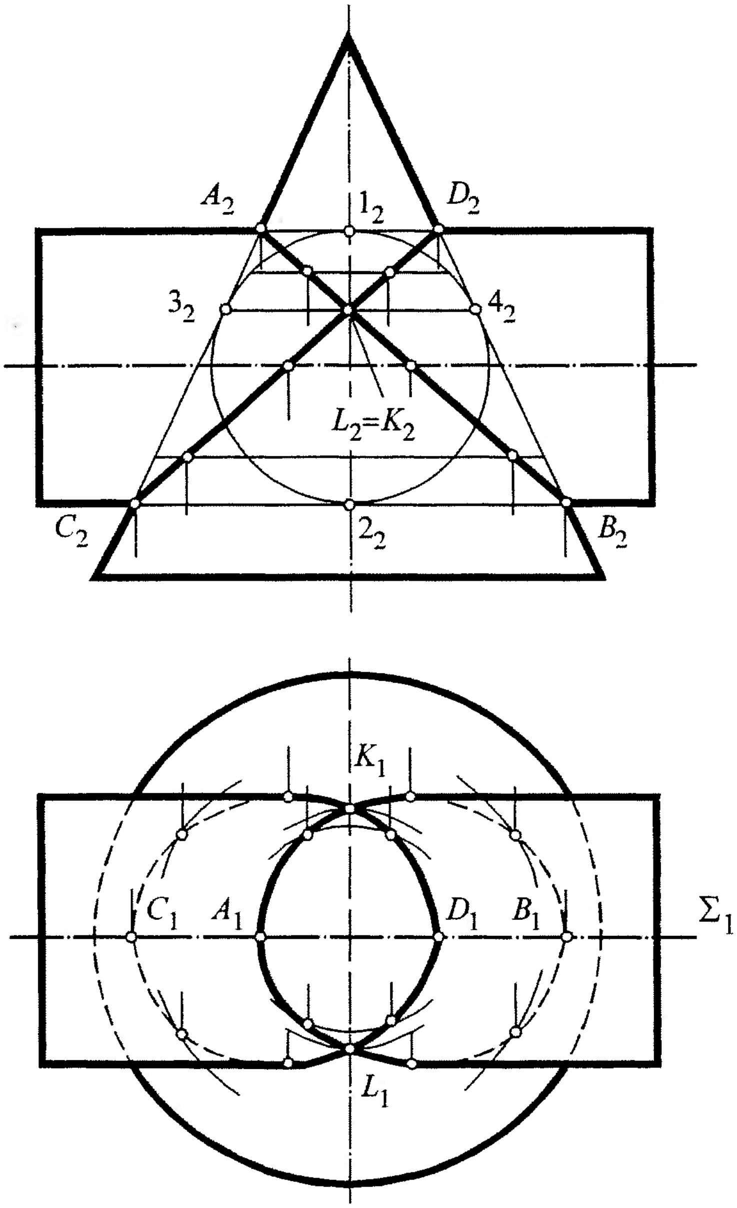


Fig. 75

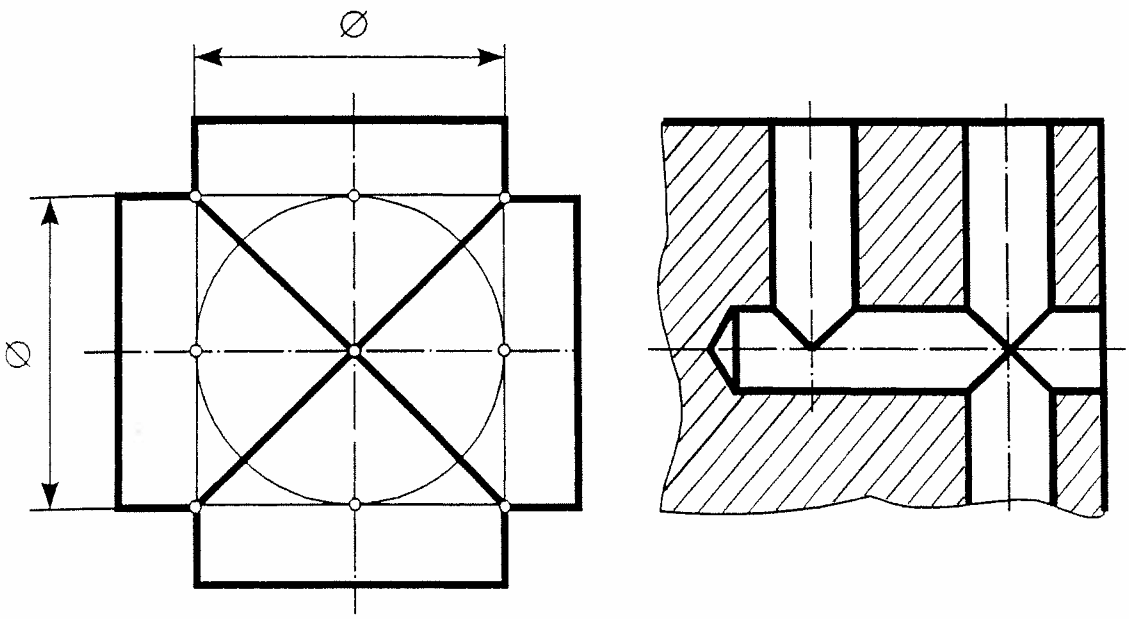

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Fig. 76

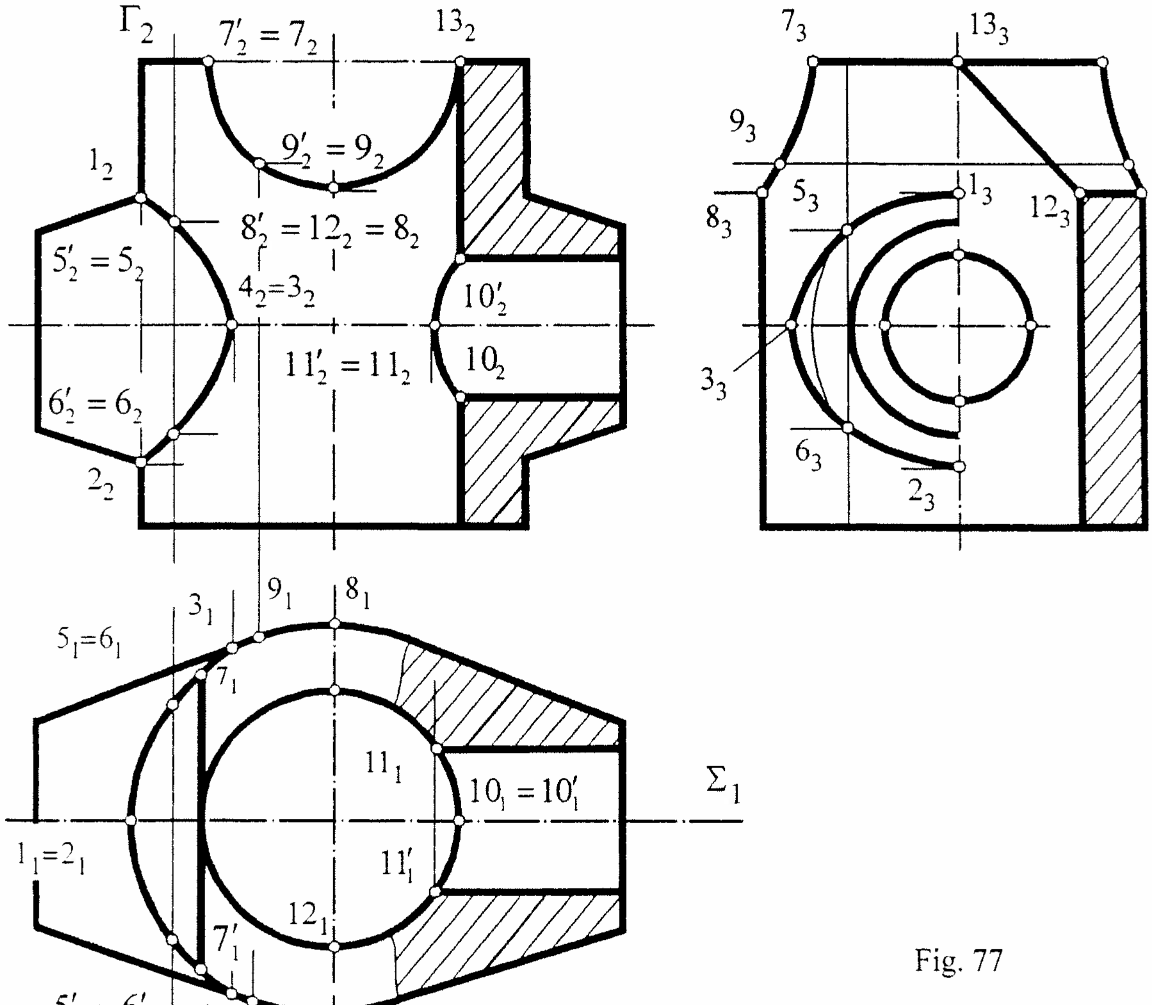
Γ_2
$7'_2 = 7_2$
13_2
1_2
$9'_2 = 9_2$
$8'_2 = 12_2 = 8_2$
$5'_2 = 5_2$
$4_2 = 3_2$
$10'_2$
$11'_2 = 11_2$
10_2
$6'_2 = 6_2$
2_2
7_3
13_3
9_3
5_3
1_3
12_3
8_3
3_3
6_3
2_3
3_1
9_1
8_1
$5_1 = 6_1$
7_1
11_1
$10_1 = 10'_1$
Σ_1
$1_1 = 2_1$
$11'_1$
12_1
$7'_1$
$5'_1 = 6'_1$
4_1
$9'_1$
$8'_1$
Γ_1

Fig. 77

Questions for self-control

1. Tell about the relative position of lines in space and in a complicated drawing.
2. What intersection lines of a sphere with a plane can be obtained?
3. What cone sections can be obtained when the plane intersects it?
4. What sections of the rotation cylinder can be obtained when the plane intersects it?
5. What is the intersection line of a polyhedron with a plane in a general case?
6. What is the intersection line of two surfaces?
7. What should be the sequence of solving problems on the construction of the intersection line of surfaces?
8. What is the gist of the Method of Auxiliary cutting planes?
9. How can the method be represented symbolically?
10. What can the intersection of surfaces be? Tell about complete and partial intersection of surfaces.
11. How should auxiliary cutting planes be chosen?
12. What methods are used for determining reference and intermediate points of the intersection line of two surfaces?
13. What is the intersection line of polyhedral and curved surfaces?
14. What are reference points of the intersection line of polyhedral and curved surfaces?
15. What is the character of the intersection line of two curved surfaces?
16. What are reference points of the intersection line of two curved surfaces?
17. What is the character of the intersection line of two polyhedral surfaces?
18. What are reference points of the intersection line of two polyhedral surfaces?
19. How is the visibility of the intersection line in the drawing defined?
20. What are coaxial revolution surfaces?
21. What are intersection lines of coaxial revolution surfaces?
22. What property of coaxial revolution surfaces is the employment of auxiliary spheres based on?
23. When can the Method of Concentric Sphere be used?
24. When can the Method of Eccentric Sphere be used?
25. What does the Monzh theorem consist in?